Axions and the Galactic Angular Momentum Distribution

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Ninth Patras Workshop on Axions,
WIMPs and WISPs
Mainz, June 26, 2013

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Outline

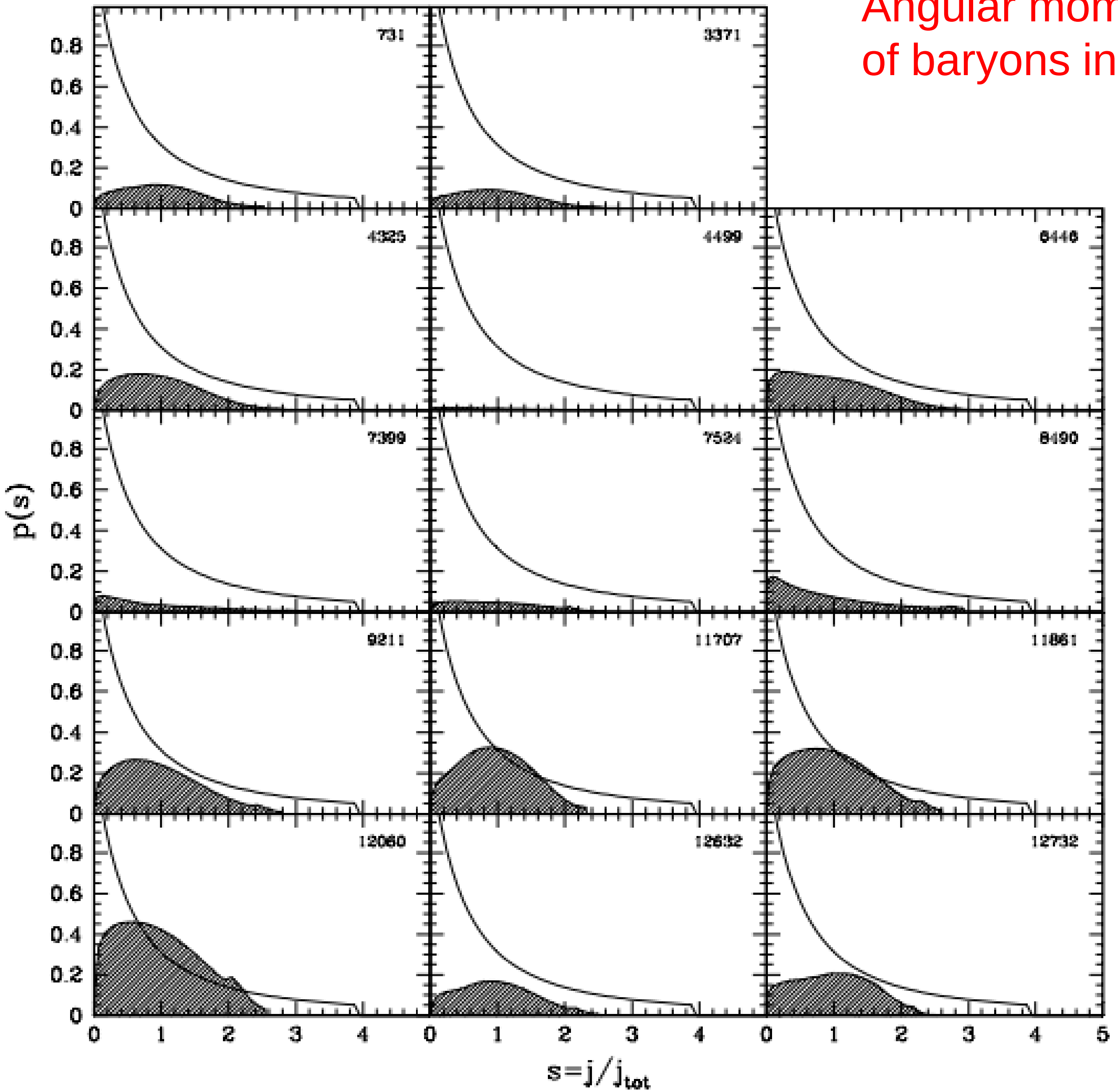
- The galactic angular momentum problem
- A critique of the isothermal halo model
- Axion Bose-Einstein condensation

axions are different

- The galactic angular momentum distribution in the axion case

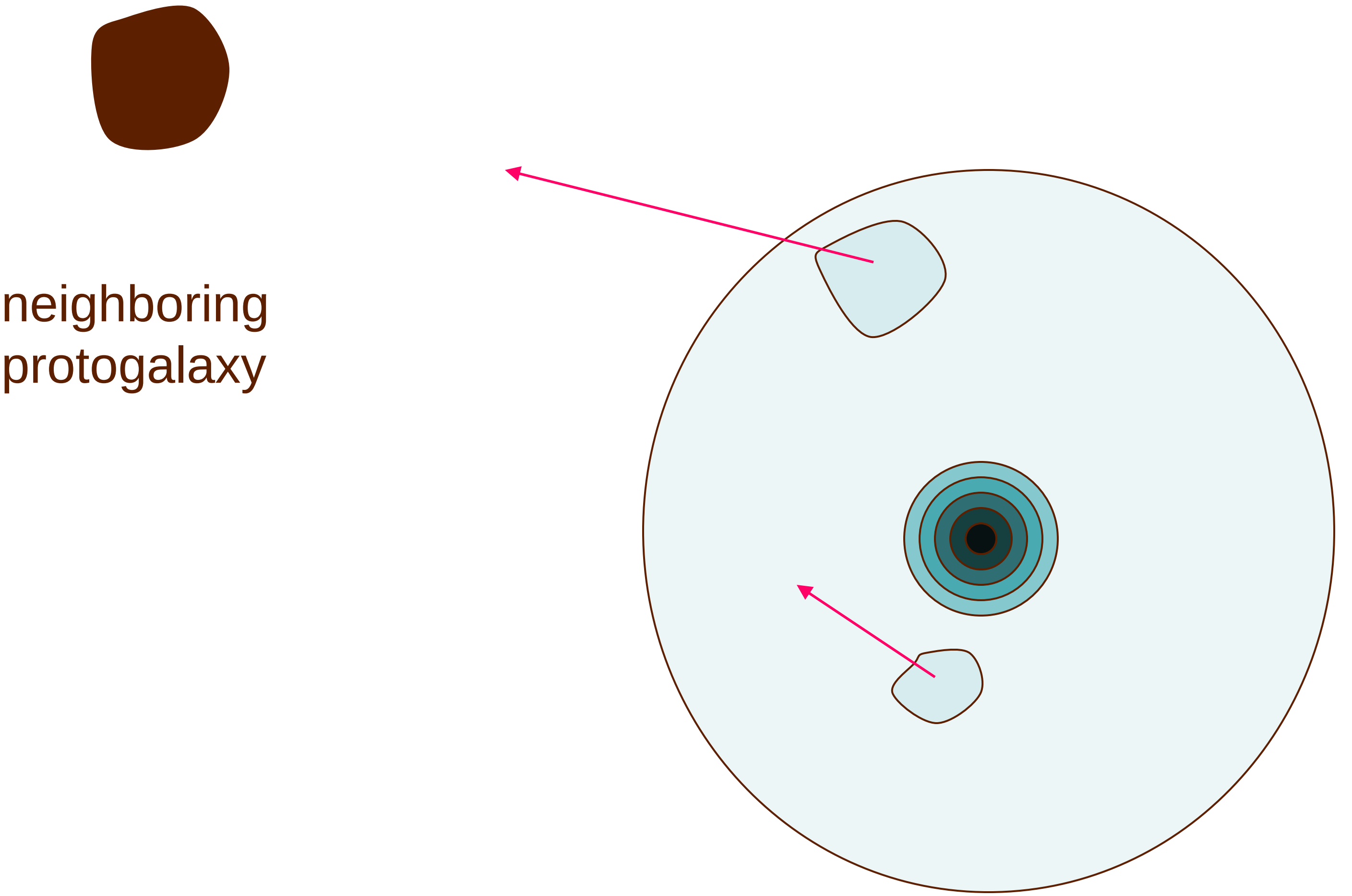
axions are better

Angular momentum distribution of baryons in dwarf galaxies



from
F. van den Bosch,
A. Burkert and
R. Swaters,
MNRAS 326
(2001) 1205

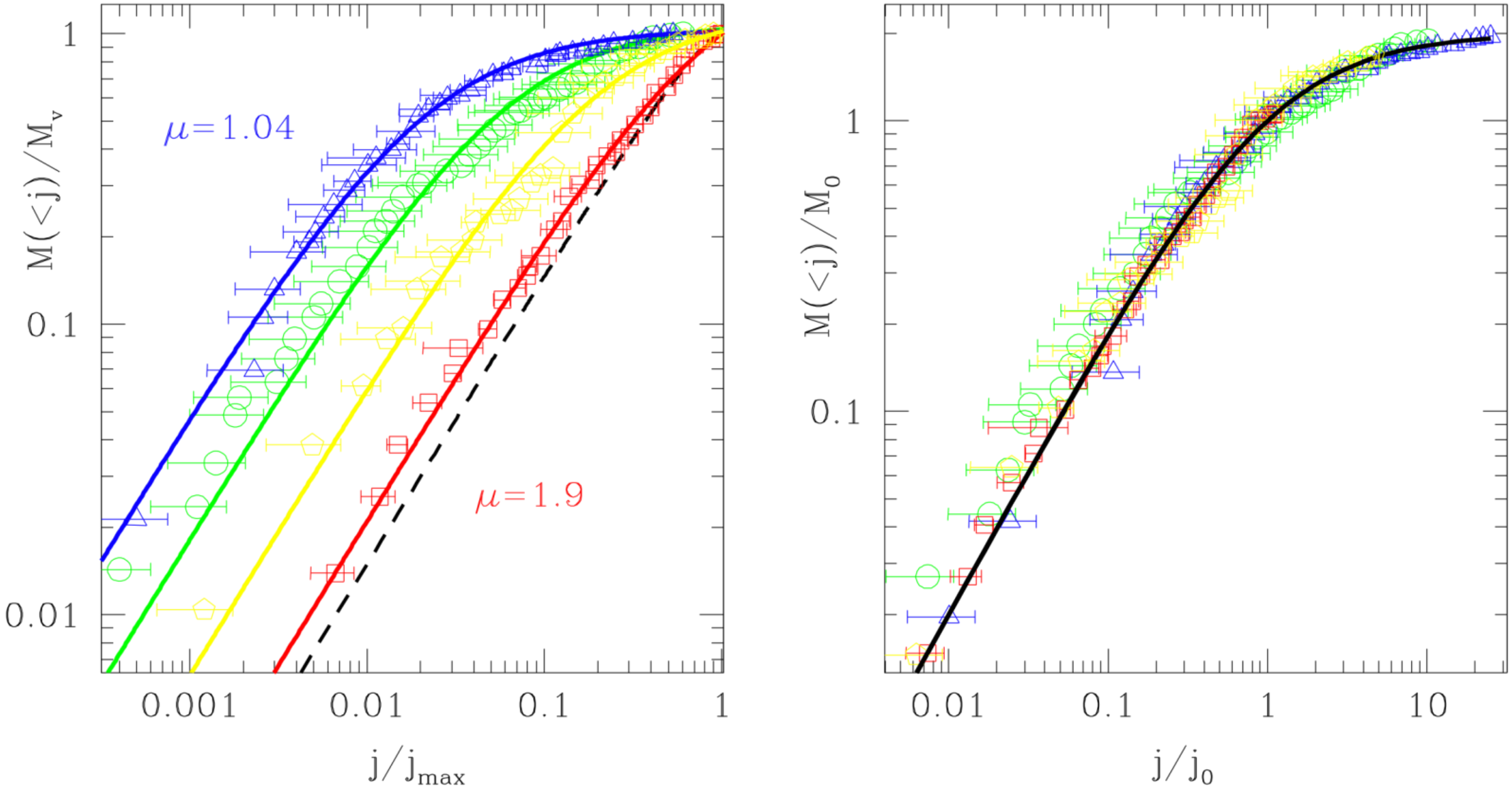
Tidal torque theory



Stromberg 1934; Hoyle 1947; Peebles 1969, 1971

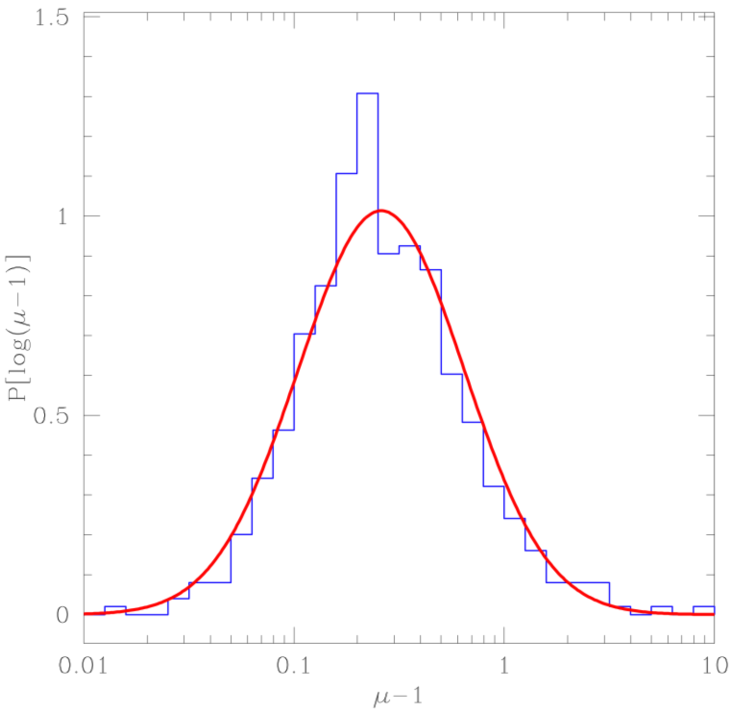
Angular momentum distribution in simulated cold dark matter halos

Bullock et al. 2001



$$\frac{dM}{dl} = \frac{\mu M_v l_0}{(l_0 + l)^2}$$

$$0 \le l \le l_{\max} = \frac{l_0}{\mu - 1}$$

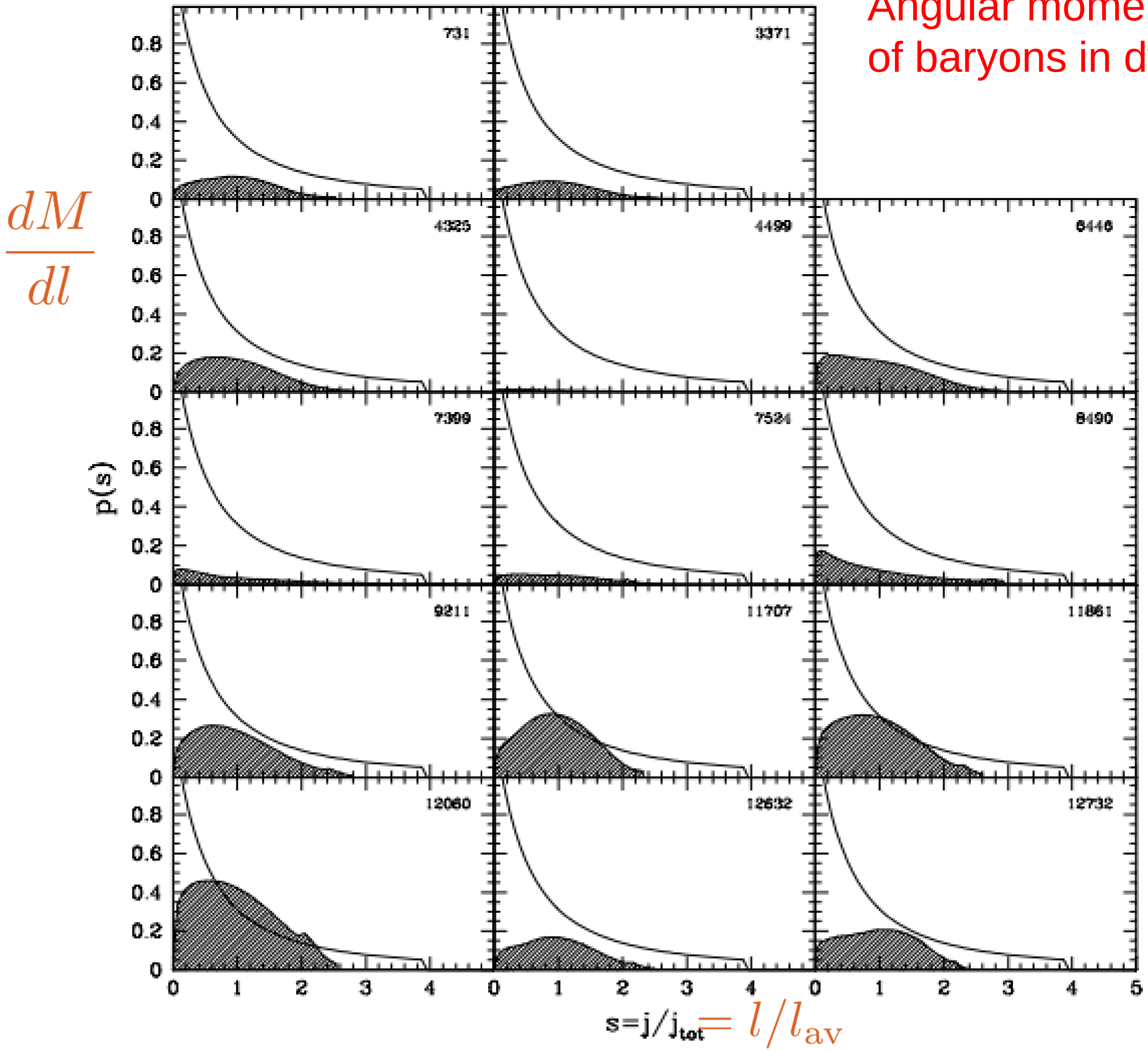


$$\frac{l_{av}}{l_{\max}} = (\mu - 1)\left[\mu \ln\left(\frac{\mu}{\mu - 1}\right) - 1\right]$$

$\frac{l_{\max}}{l_{av}}$ has 90% range 2.6 - 8.1

median 4.0

Angular momentum distribution of baryons in dwarf galaxies



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The angular momentum distribution of CDM in simulations differs from that of baryons in dwarf galaxies

1) the shape is different

2) $$\frac{l_{\rm max}}{l_{\rm av}} \simeq 2.6$$ observed

whereas

$$2.6 < \frac{l_{\rm max}}{l_{\rm av}} < 8.1$$ in simulations

Processes that allow angular momentum exchange aggravate the discrepancy rather than resolve it

- Frictional forces among the baryons have the general effect of removing angular momentum from baryons that have little and transferring it to baryons that have a lot.

- Dynamical friction of dark matter on clumps of baryonic matter has the general effect of transferring angular momentum from the baryons to the dark matter.

-> GALACTIC ANGULAR MOMENTUM PROBLEM

Navarro and Steinmetz 2000
Burkert and D'Onglia 2004

Self-gravitating Isothermal Sphere

(S. Chandrasekar, 1939)

$$\mathcal{N}(\vec{r},\vec{v}) = \mathcal{N}_0 e^{-\frac{m}{T}\left[\frac{1}{2}\vec{v}\cdot\vec{v}+\Phi(r)\right]}$$

$$\nabla^2\Phi(r) = 4\pi G m n(r) = 4\pi G m \int d^3r\,\mathcal{N}(\vec{r},\vec{v})$$

as a model of galactic halos

has many virtues ...

$$n(r) \simeq \frac{n(o)a^2}{r^2 + a^2}$$

1) depends on only two parameters

2) the two parameters are determined in terms of observable properties of the galaxy, its rotation velocity and core radius

3) predicts asymptotically flat rotation curves, in agreement with observation

4) predicts approximately constant density at galactic centers, also in agreement with observation

5) follows from a simple physical principle, thermalization

Nonetheless, as a model of galactic halos, the isothermal sphere is flawed

1) there is no justification for assuming thermal equilibrium.

J. Binney and S. Tremaine, 1987, p 601

If for some unexplained reason, the Milky Way halo were in thermal equilibrium today, it would soon leave thermal equilibrium because it accretes additional dark matter that does not thermalize over the age of the universe.

The infalling dark matter produces discrete flows and caustics.

J. Ipser and P.S. 1992

2) the isothermal sphere model does not allow the halo to have any angular momentum.

Thermal distribution of a many-body system

$$\mathcal{N}_i = \frac{1}{e^{\frac{1}{T}(\epsilon_i - \mu)} - \sigma}$$

$$\sigma = 0, \quad +1, \quad -1$$

Maxwell-Boltzmann

Bose-Einstein

Fermi-Dirac

$$T = \text{temperature}$$

$$\mu = \text{chemical potential}$$

Thermal distribution of a rotating many-body system

$$\mathcal{N}_i = \frac{1}{e^{\frac{1}{T}(\epsilon_i - \mu - \omega l_i)} - \sigma}$$

$$\sigma = 0, \quad +1, \quad -1$$

Maxwell-Boltzmann

Bose-Einstein

Fermi-Dirac

$T = \text{temperature}$

$\mu = \text{chemical potential}$

$\omega = \text{angular velocity}$

Rotating isothermal

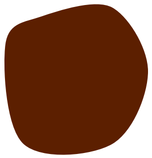
$$\mathcal{N}(\vec{r},\vec{v}) = \mathcal{N}_0 e^{-\frac{m}{T}\left[\frac{1}{2}\vec{v}\cdot\vec{v}+\Phi(\vec{r})-\omega\hat{z}\cdot(\vec{r}\times\vec{v})\right]}$$

$$= \mathcal{N}_0 e^{-\frac{m}{T}\left[\frac{1}{2}(\vec{v}-\omega\hat{z}\times\vec{r})^2+\Phi(\vec{r})-\frac{1}{2}\omega^2\rho^2\right]}$$

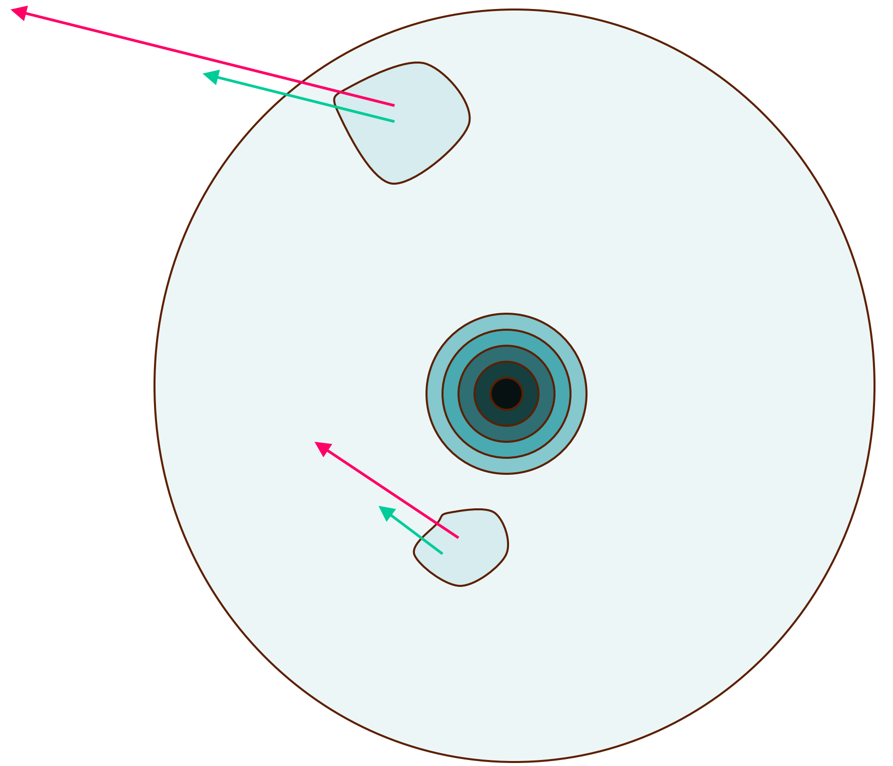
$$n(\rho,z) = n_0 e^{\frac{m}{T}\left[\frac{1}{2}\omega^2\rho^2-\Phi(\rho,z)\right]}$$

poor description of galactic halos
as soon as ω differs from zero

Tidal torque theory with ordinary CDM



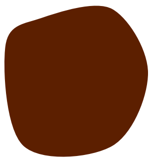
neighboring protogalaxy

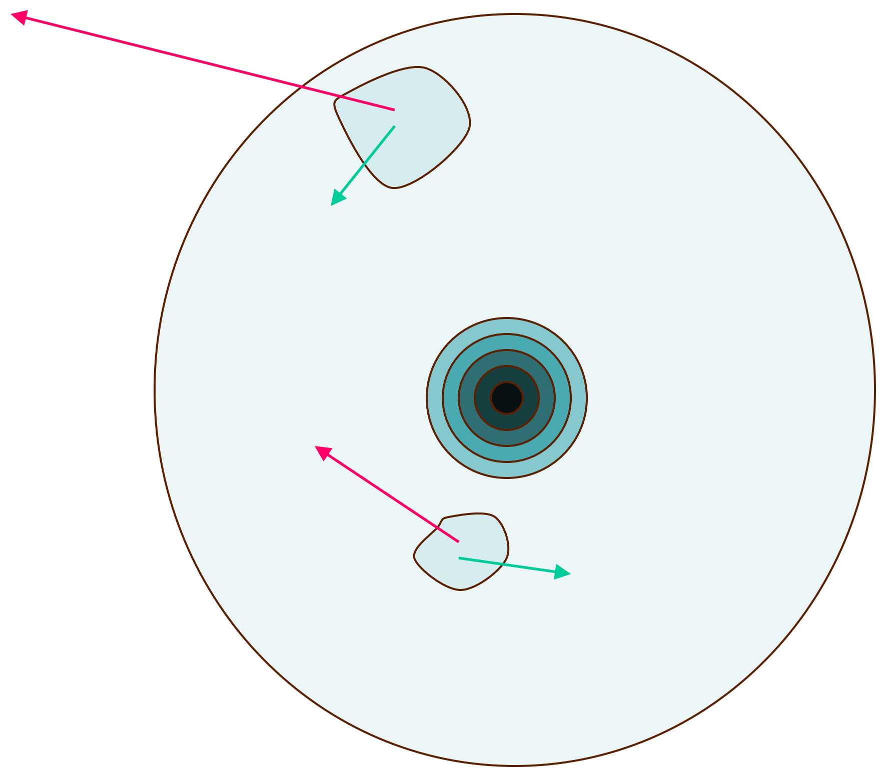


$$\vec{\nabla} \times \vec{v} = 0$$

the velocity field remains irrotational

Tidal torque theory with axion BEC

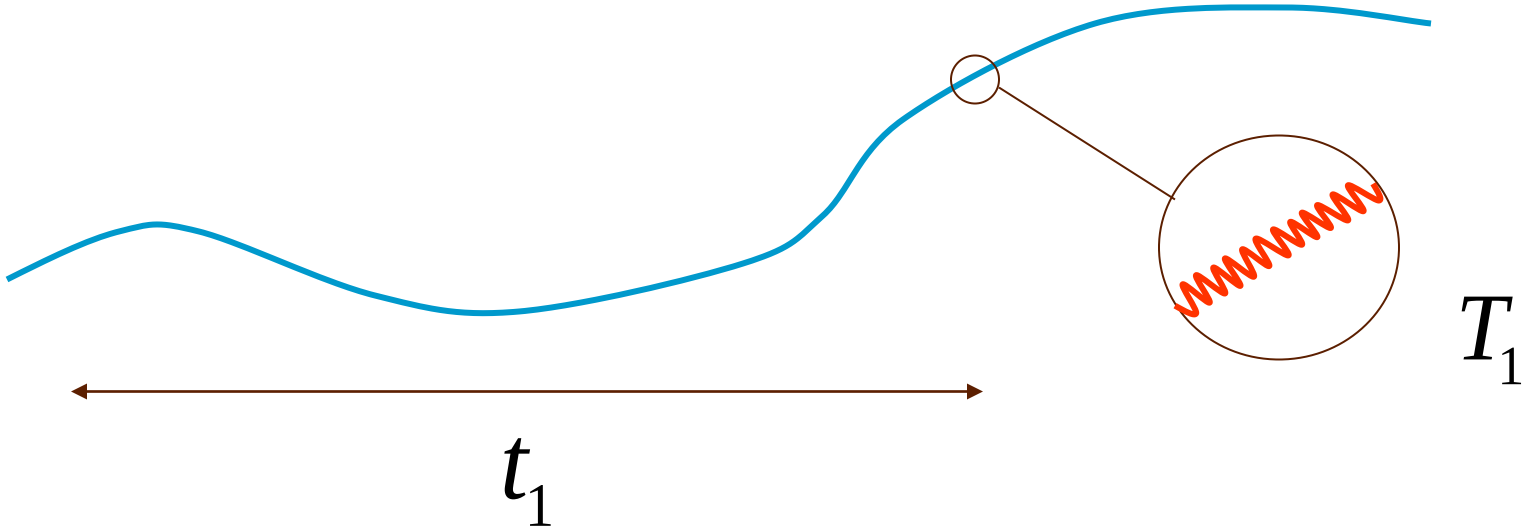




$$\vec{\nabla}\times\vec{v}\neq 0$$

in their lowest energy available state, the axions fall in with net overall rotation

There are two cosmic axion populations: hot and cold.



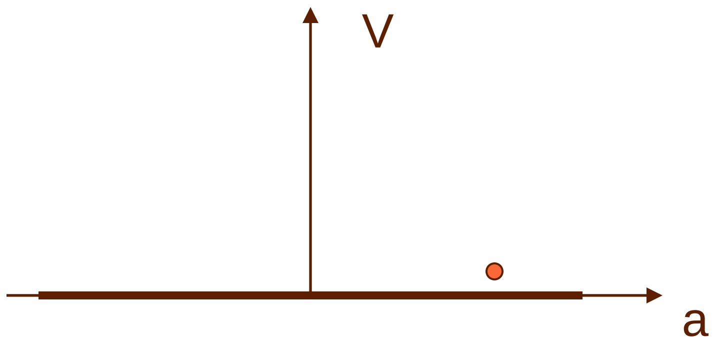
When the axion mass turns on, at QCD time,

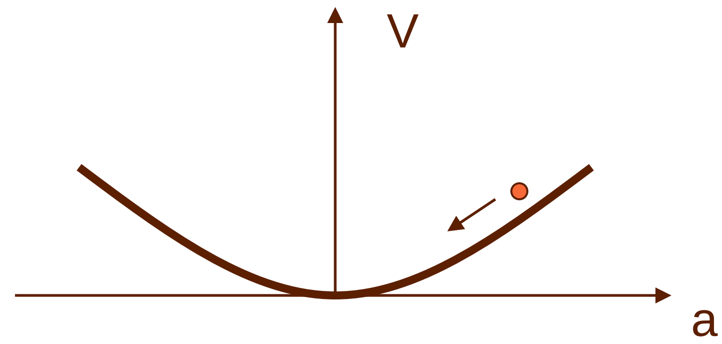
$T_1 \sim 1\ \text{GeV}$

$t_1 \sim 2 \cdot 10^{-7}\ \text{sec}$

$$p_a(t_1) = \frac{1}{t_1} \sim 3 \cdot 10^{-9}\ \text{eV}$$

Axion production by vacuum realignment





$$T \geq 1\,\mathrm{GeV} \qquad\qquad T \leq 1\,\mathrm{GeV}$$

$$n_a(t_1) \sim \frac{1}{2}\, m_a(t_1)\, a(t_1)^2 \sim \frac{1}{2t_1}\, f_a^{\,2}\, \alpha(t_1)^2$$

initial misalignment angle

$$\rho_a(t_0) \sim m_a\, n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a^{-\frac{7}{6}}$$

J. Preskill, F. Wilczek + M.Wise; L. Abbott + P.S.; M. Dine + W. Fischler 1983

Cold axion properties

- number density

$$n(t) \sim \frac{4\cdot 10^{47}}{\mathrm{cm}^3}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{\frac{5}{3}}\left(\frac{a(t_1)}{a(t)}\right)^3$$

- velocity dispersion

$$\delta \mathrm{v}(t) \sim \frac{1}{m_a\, t_1}\,\frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\mathcal{N} \sim n(t)\,\frac{(2\pi)^3}{\frac{4\pi}{3}(m_a\,\delta \mathrm{v})^3} \sim 10^{61}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{\frac{8}{3}}$$

Bose-Einstein Condensation

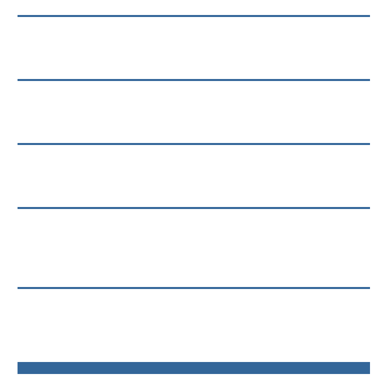
if identical bosonic particles
are highly condensed in phase space
and their total number is conserved
and they thermalize

then most of them go to the lowest energy
available state

why do they do that?

by yielding their energy to the non-condensed particles, the total entropy is increased.

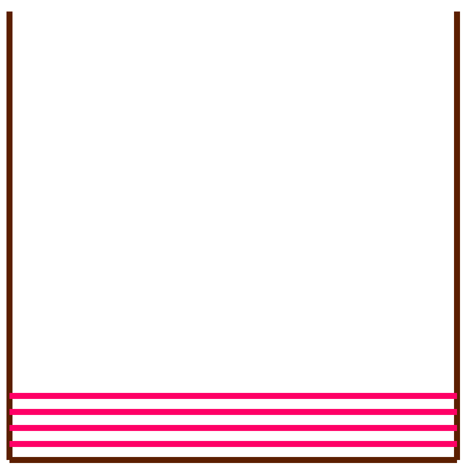
preBEC



BEC

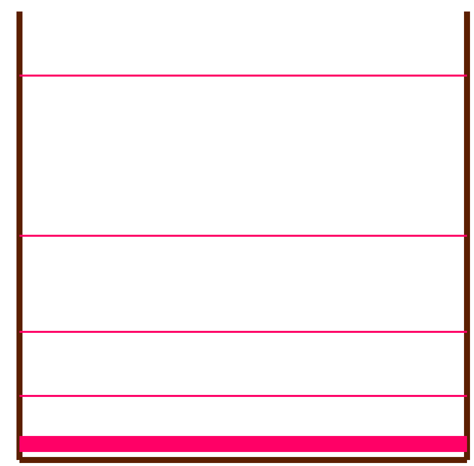
the axions thermalize and form a BEC after a time τ

$t < \tau$



the axion fluid obeys
classical field equations,
behaves like CDM

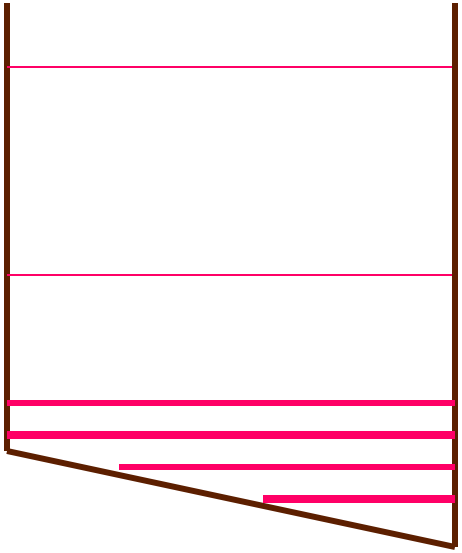
$t > \tau$



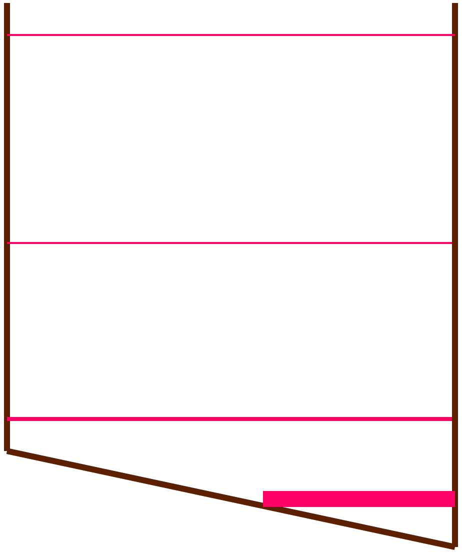
the axion fluid does not obey
classical field equations,
does not behave like CDM

the axion BEC rethermalizes

$t < \tau$



$t > \tau$



the axion fluid obeys
classical field equations,
behaves like CDM

the axion fluid does not obey
classical field equations,
does not behave like CDM

Axion field dynamics

$$H = \sum_j \omega_j a_j^\dagger a_j \quad + \quad \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

O. Erken et al.,
PRD 85 (2012)
063520

From $\frac{1}{4!}\lambda\phi^4$ self-interactions

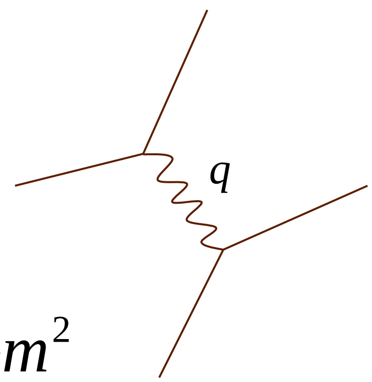
$$\Lambda_\lambda \begin{smallmatrix} \vec{p}_3, \vec{p}_4 \\ \vec{p}_1, \vec{p}_2 \end{smallmatrix} = -\frac{\lambda}{4m^2 V}\, \delta_{\vec{p}_1+\vec{p}_2, \vec{p}_3+\vec{p}_4}$$

From gravitational self-interactions

$$\Lambda_g \begin{smallmatrix} \vec{p}_3, \vec{p}_4 \\ \vec{p}_1, \vec{p}_2 \end{smallmatrix} = -\frac{4\pi G m^2}{V}\, \delta_{\vec{p}_1+\vec{p}_2, \vec{p}_3+\vec{p}_4} \left(\frac{1}{|\vec{p}_1 - \vec{p}_3|^2} + \frac{1}{|\vec{p}_1 - \vec{p}_4|^2} \right)$$

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301



$$\frac{Gm^2}{q^2}$$

$$\Gamma_g \sim 4\pi G n m^2 l^2 \qquad \text{with } l = (m\,\delta \mathrm{v})^{-1}$$

$$\sim 5 \cdot 10^{-7} H(t_1) \left(\frac{f}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}}$$

at time t_1

$$\Gamma_g(t)/H(t) \propto t\, a(t)^{-1} \propto a(t)$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \sim 500 \text{ eV} \left(\frac{f}{10^{12} \text{ GeV}} \right)^{\frac{1}{2}}$$

After that

$$\delta \mathrm{v} \sim \frac{1}{m\,t}$$

$$\Gamma_g(t) / H(t) \propto t^3 \, a(t)^{-3}$$

Axions rethermalize before falling onto galactic halos and go to their lowest energy state consistent with the total angular momentum they acquired from tidal torquing

provided $$4\pi G n m^2 \ell > \ m\dot{v}$$

i.e. $$nm > \frac{1}{30}\,\rho_{\mathrm{DM}}$$

Axion fraction of dark matter is more than of order 3%.

The lowest energy available state is one in which each spherical shell rotates rigidly with angular velocity

$$\omega(r) \propto \frac{1}{r^2}$$

On the turnaround sphere, the angular momentum distribution is

$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n}\times(\hat{z}\times\hat{n}) \; \frac{R(t)^2}{t}$$

net overall rotation

The vortices in the axion BEC are attractive and join into a big vortex

The infall rate

$$\frac{dM}{d\Omega dt}(\theta, t) = N_v (\sin\theta)^v \frac{M(t)}{2\pi t}$$

is not isotropic.

$$\frac{N_v}{4\pi} \int d\Omega \ (\sin\theta)^v = 1$$

Baryons and WIMPs are entrained by the axion BEC

$$4\pi G n m m' \ell > \; m' \dot{v}$$

is the same condition as

$$4\pi G n m^2 \ell > \; m \dot{v}$$

i.e. $$nm > \frac{1}{30} \, \rho_{\rm dm}$$

Baryon/WIMP specific angular momentum distribution on the turnaround sphere

$$\vec{\ell}(\hat{n},t) = j_{\max}\ \hat{n}\times(\hat{z}\times\hat{n})\ \frac{R(t)^2}{t}$$

and infall rate

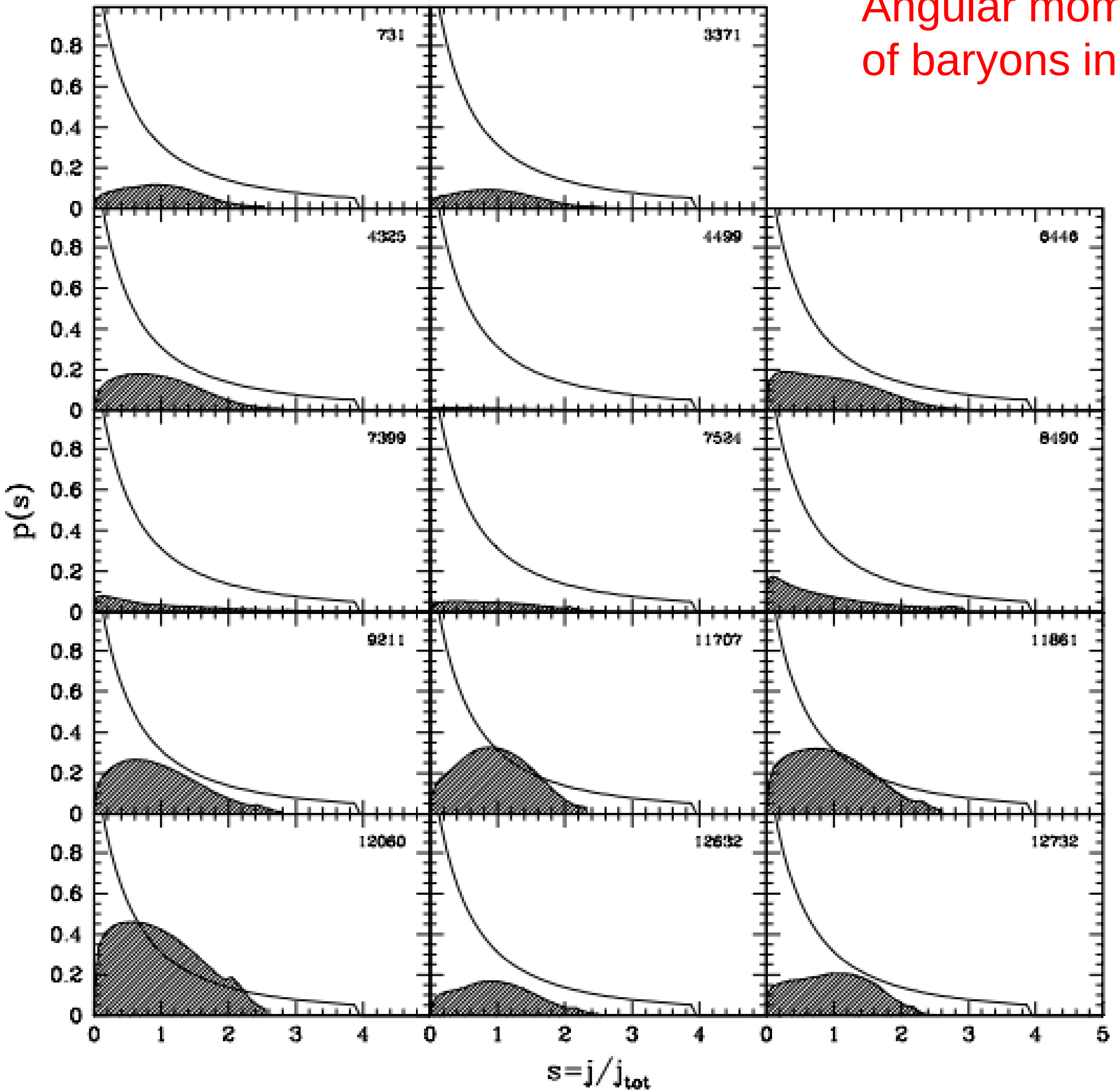
$$\frac{dM}{d\Omega dt}(\theta,t) = N_{\upsilon'}(\sin\theta)^{\upsilon'}\frac{M(t)}{2\pi t}$$

$$l(\theta, t) = l_{\max} \, (\sin\theta)^2 \left(\frac{t}{t_0}\right)^{\frac{5}{3}}$$

$$\frac{dM}{d\Omega dt}(\theta, t) = N_{\upsilon'} (\sin\theta)^{\upsilon'} \frac{M(t)}{2\pi t}$$

$$\frac{dM}{dl}(l) = \int d\Omega \int_0^{t_0} dt \; \frac{dM}{d\Omega dt}(\theta, t) \; \delta(l - l(\theta, t))$$

Angular momentum distribution of baryons in dwarf galaxies



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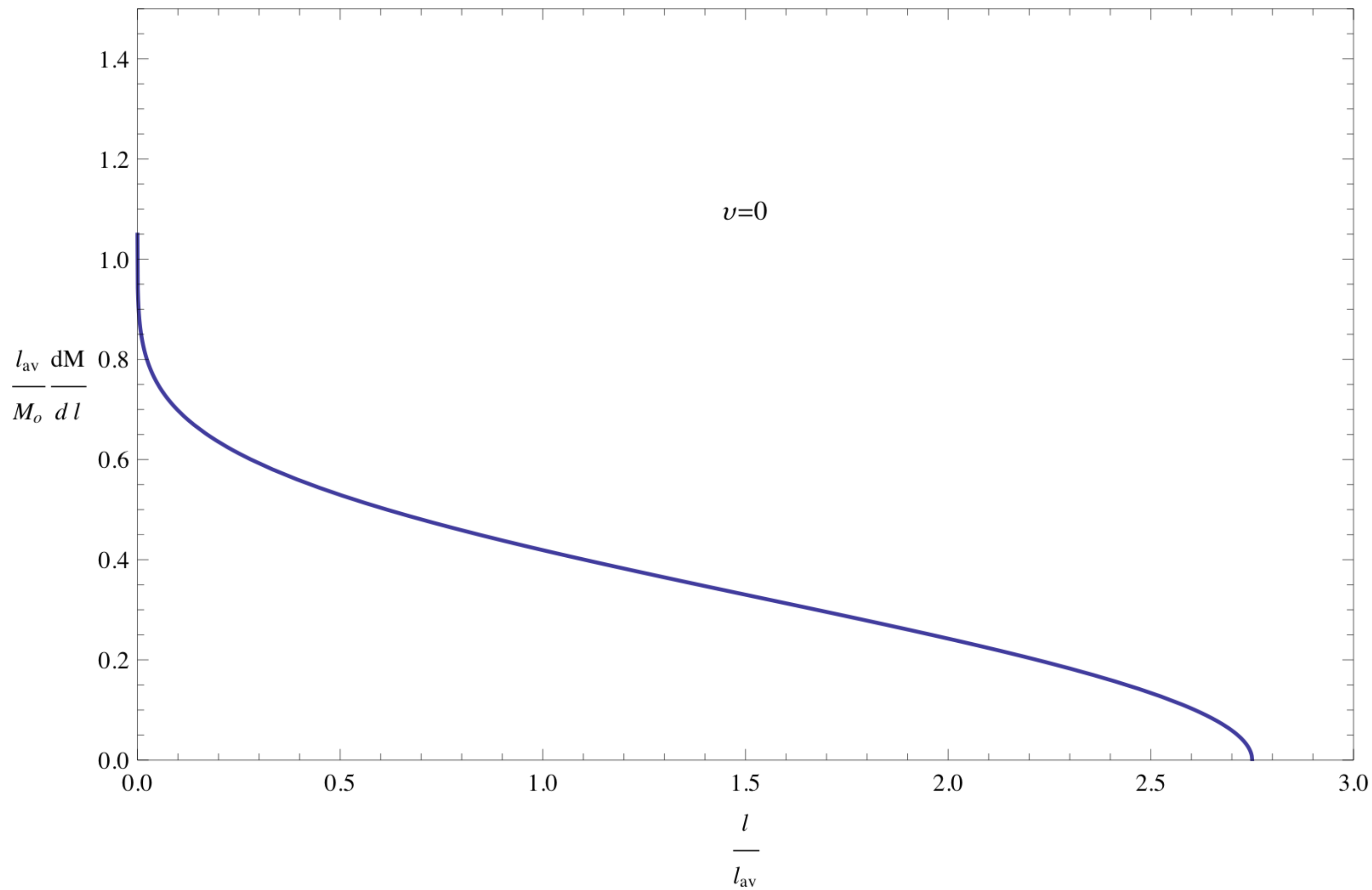
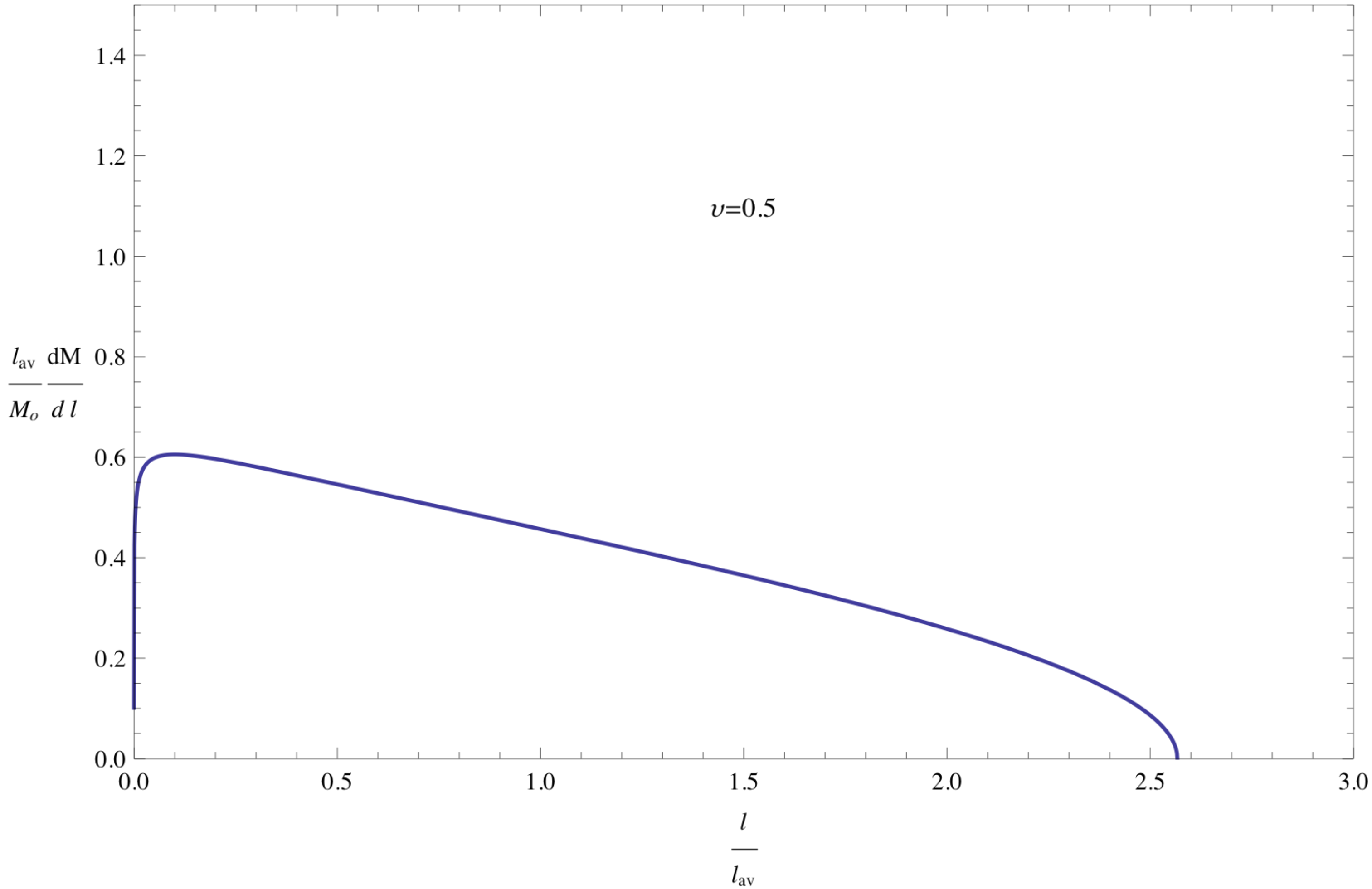
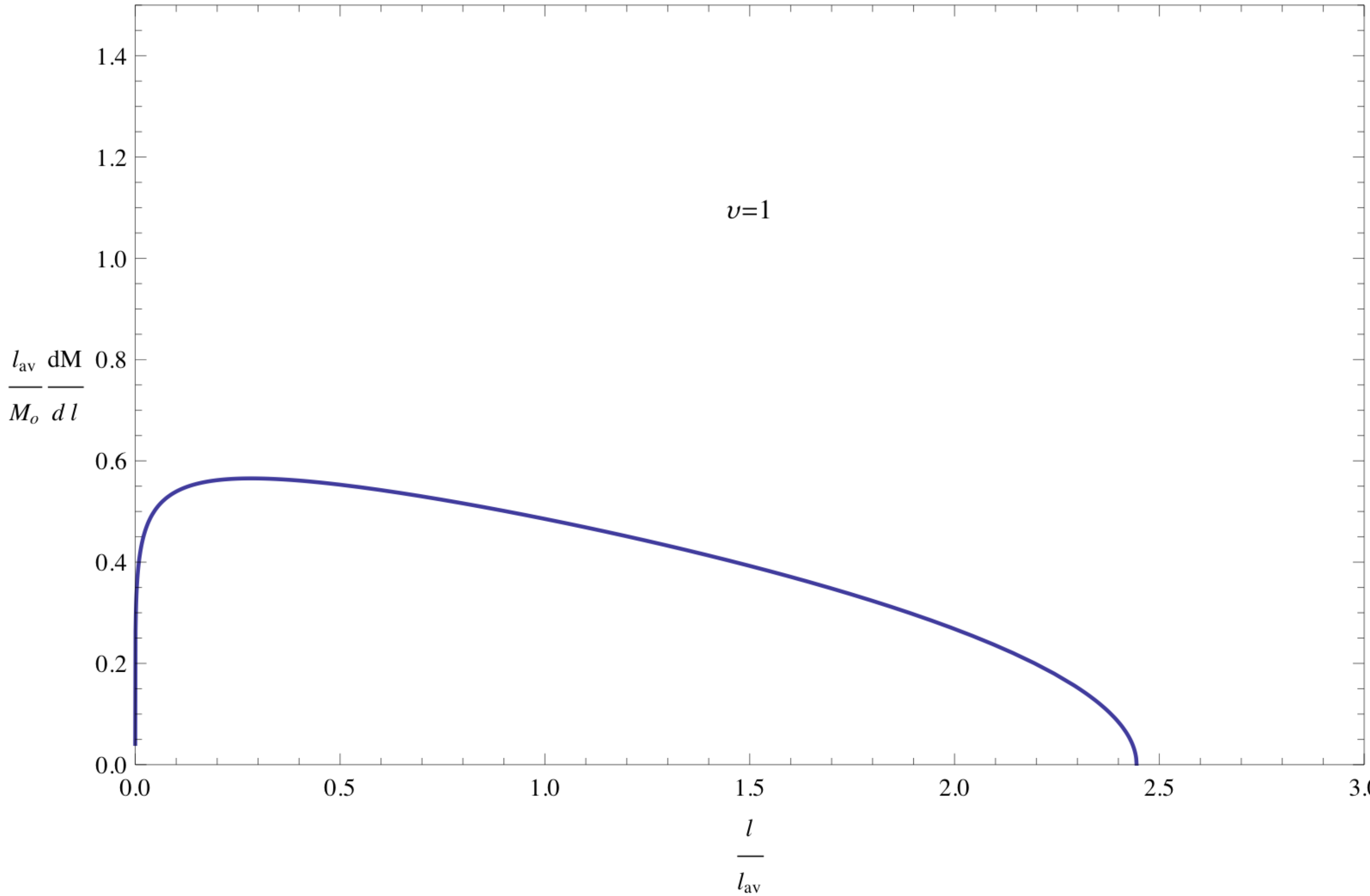


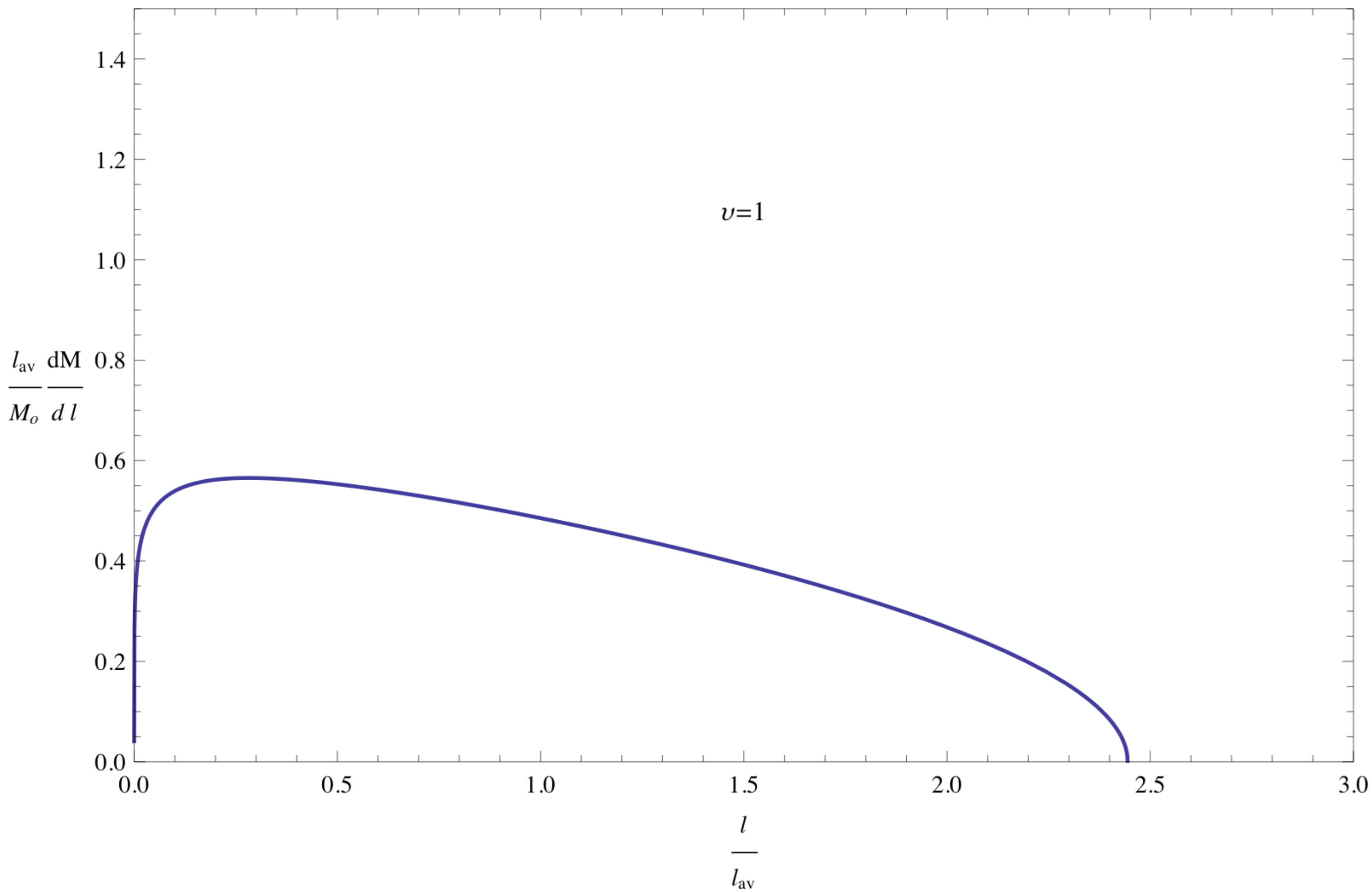
ν=0
$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
$\frac{l}{l_{av}}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.5
1.5
2.0
2.5
3.0



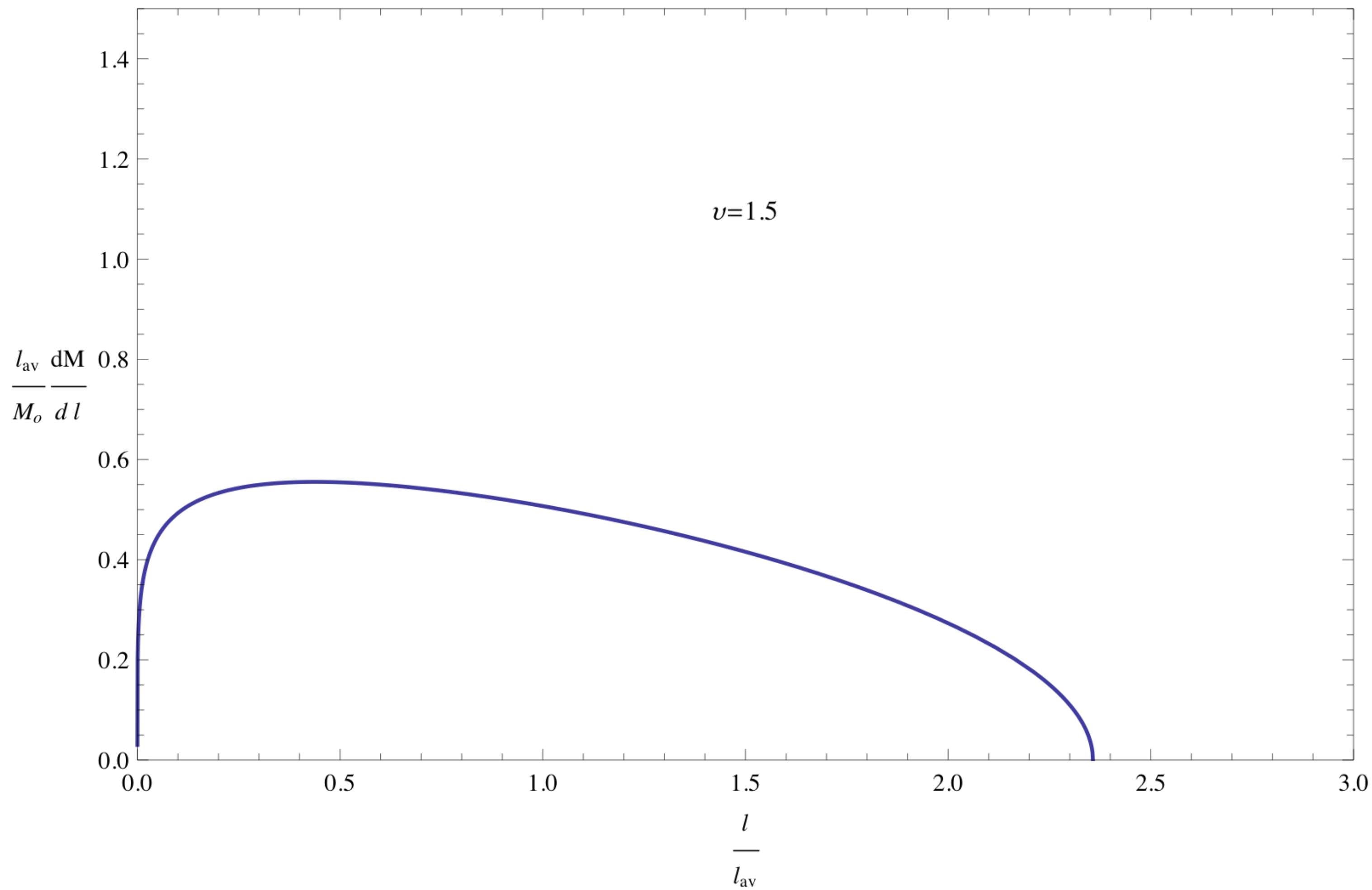
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
υ=0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\frac{l}{l_{av}}$

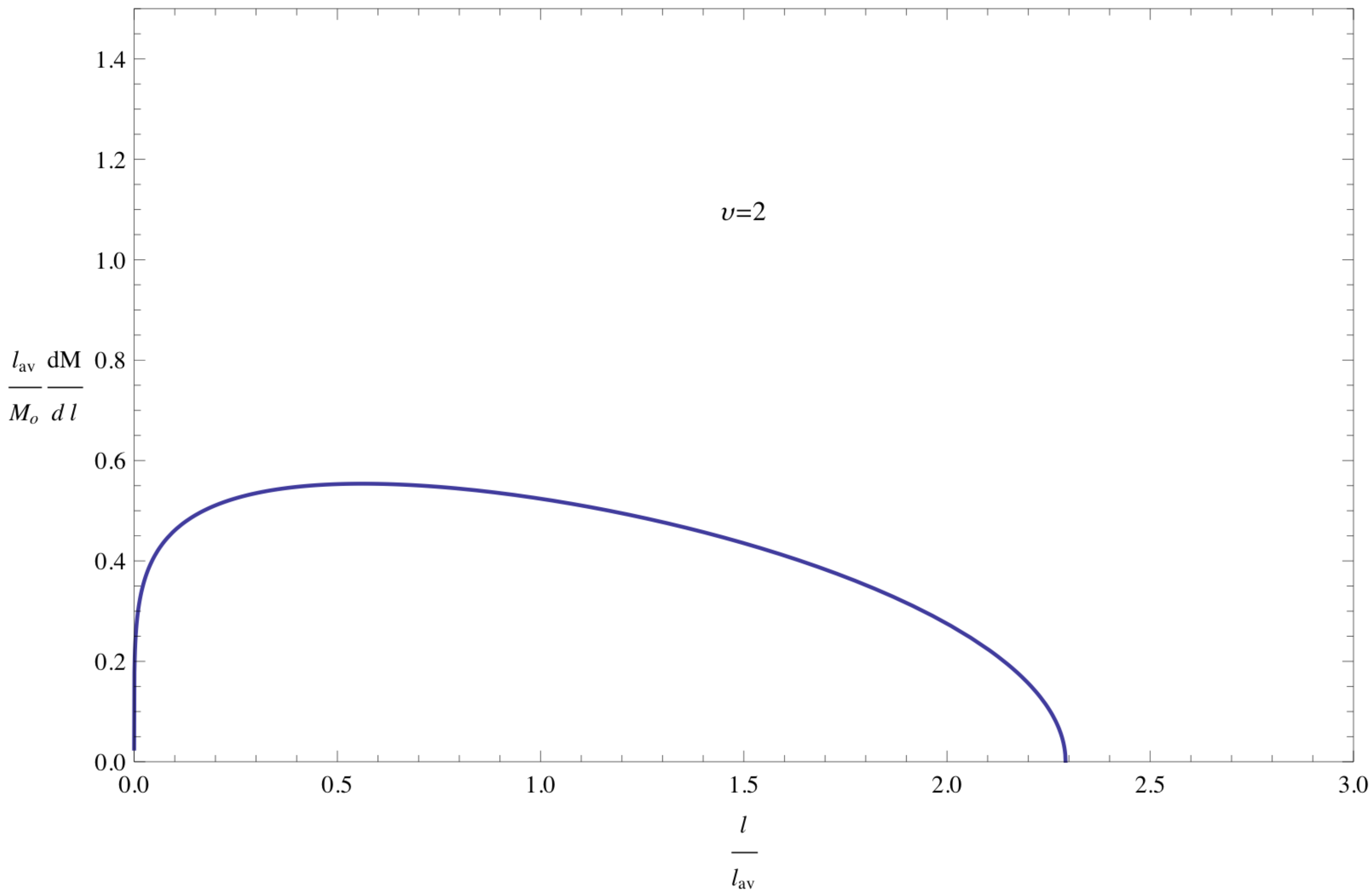


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{l_{av}}{M_o} \frac{dM}{d\,l}$
υ=1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\frac{l}{l_{av}}$

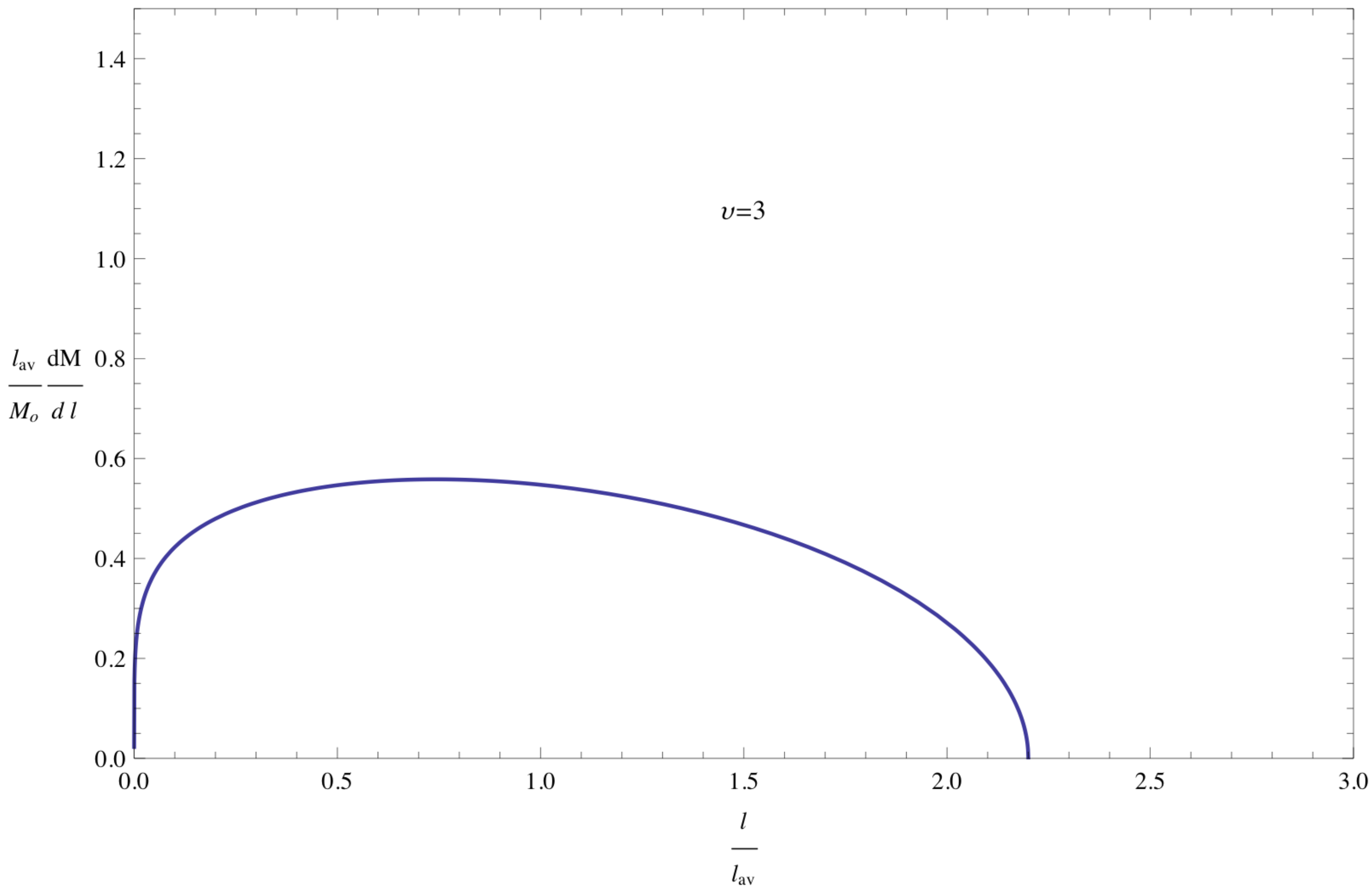


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
υ=1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\frac{l}{l_{av}}$

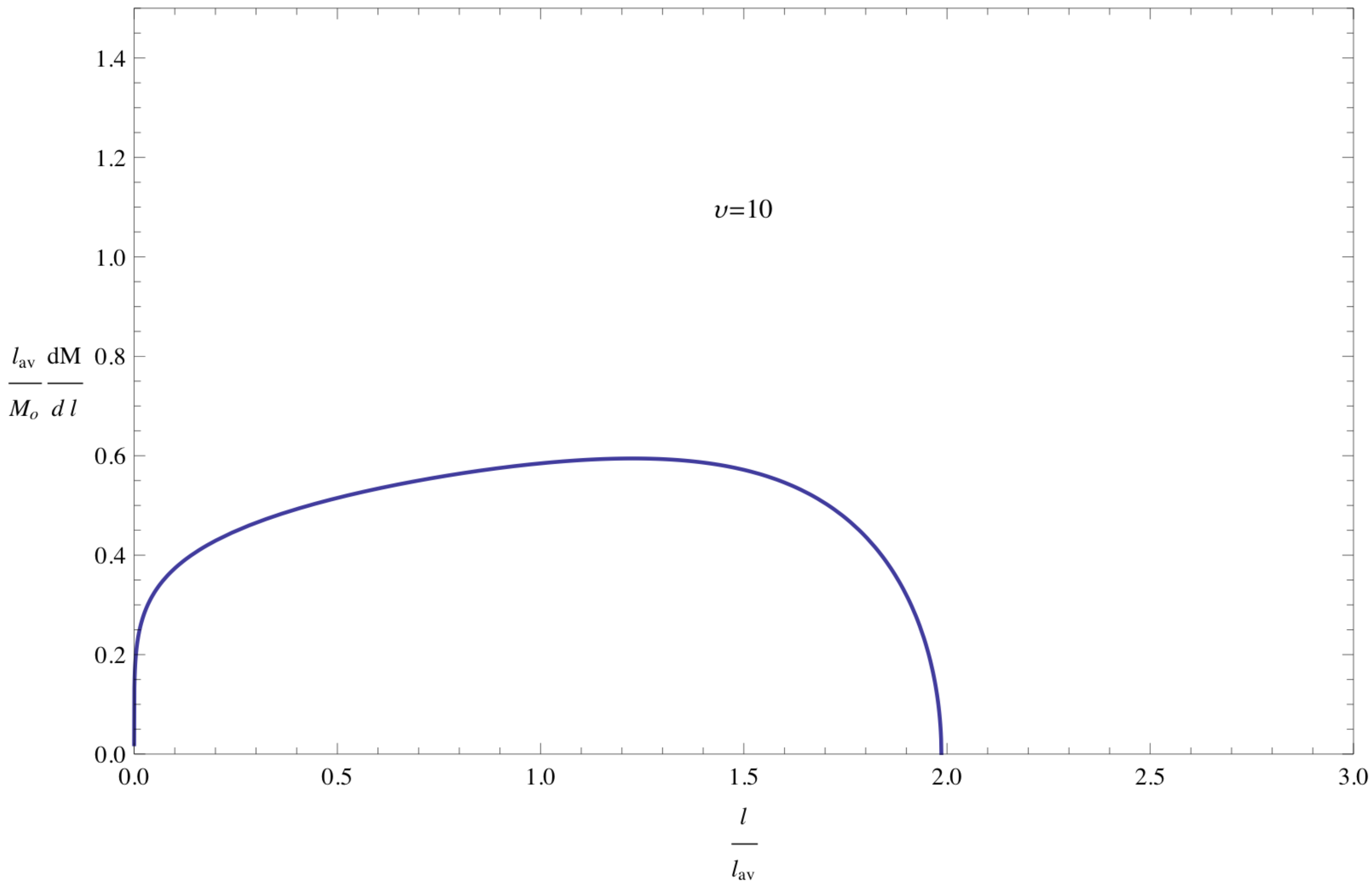

$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
υ=1.5
$\frac{l}{l_{av}}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.5
1.0
1.5
2.0
2.5
3.0



υ=2
$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
$\frac{l}{l_{av}}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0

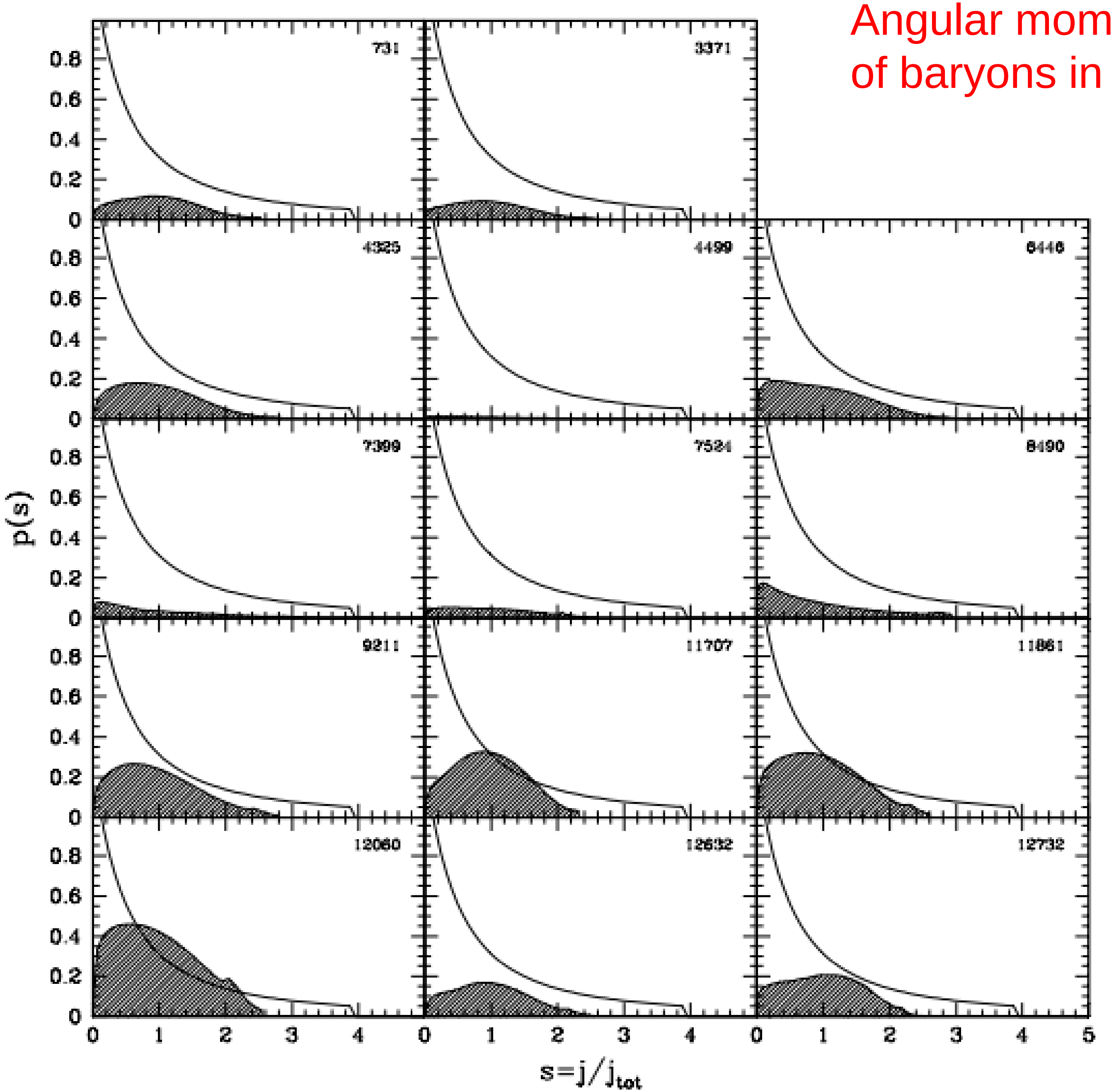


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
υ=3
$\frac{l_{av}}{M_o}\frac{dM}{d\,l}$
$\frac{l}{l_{av}}$



υ=10
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.5
2.0
2.5
3.0
$\frac{l_{av}}{M_o}\frac{dM}{dl}$
$\frac{l}{l_{av}}$

Angular momentum distribution of baryons in dwarf galaxies



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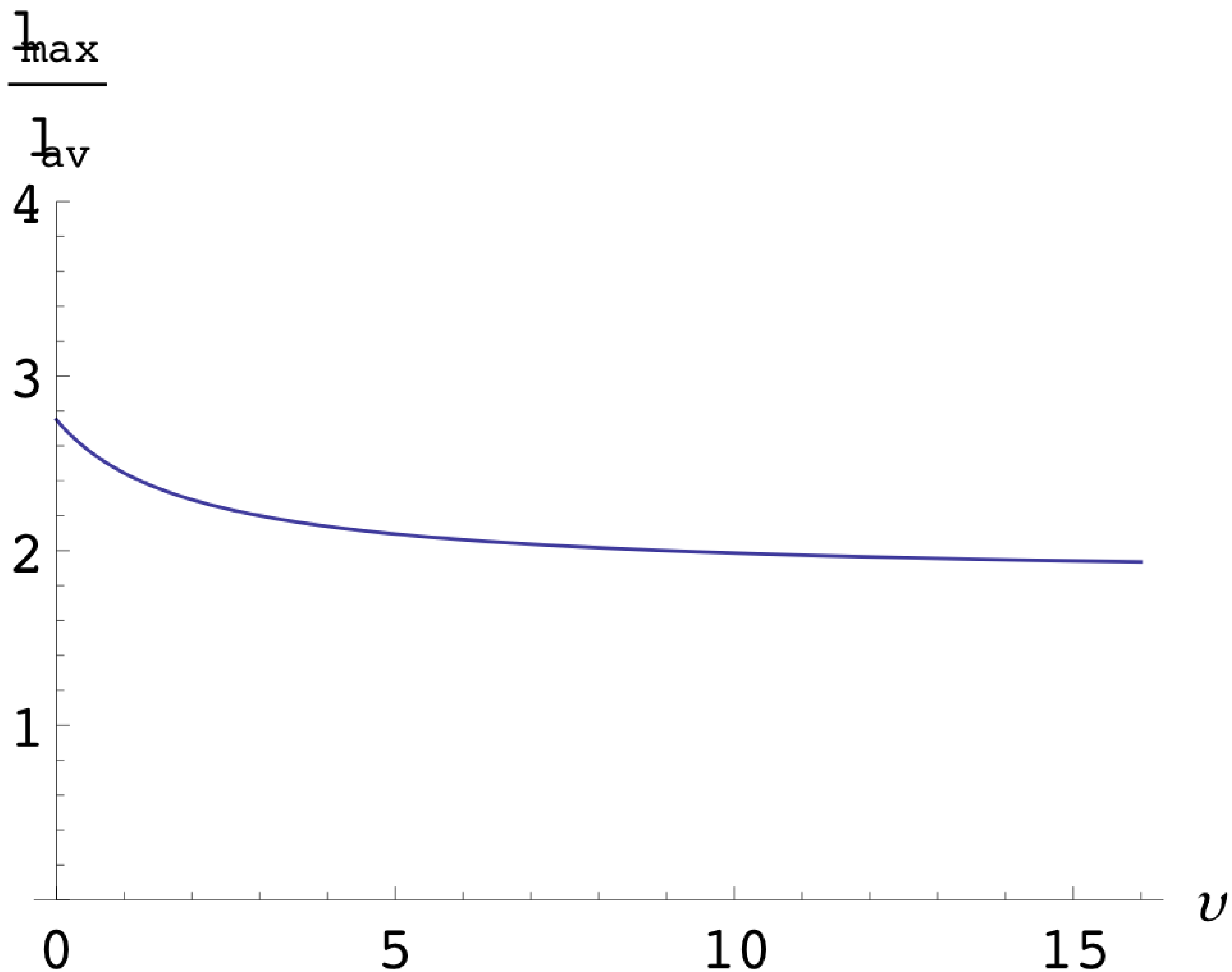

$\frac{l_{max}}{l_{av}}$
4
3
2
1
0
5
10
15
υ

Conclusions:

The galactic angular momentum problem is solved if the dark matter is axions.

The required minimum dark matter fraction in axions is of order 3%.

This evidence for axion dark matter is in addition to, and consistent with, that from the study of caustics.