9th PATRAS Workshop on Axions, WIMPs and WISPs,
Schloss Waldthausen, Mainz, Germany, 27 June, 2013

Extended axion electrodynamics, relic axions and dark matter fingerprints in the terrestrial electromagnetic field

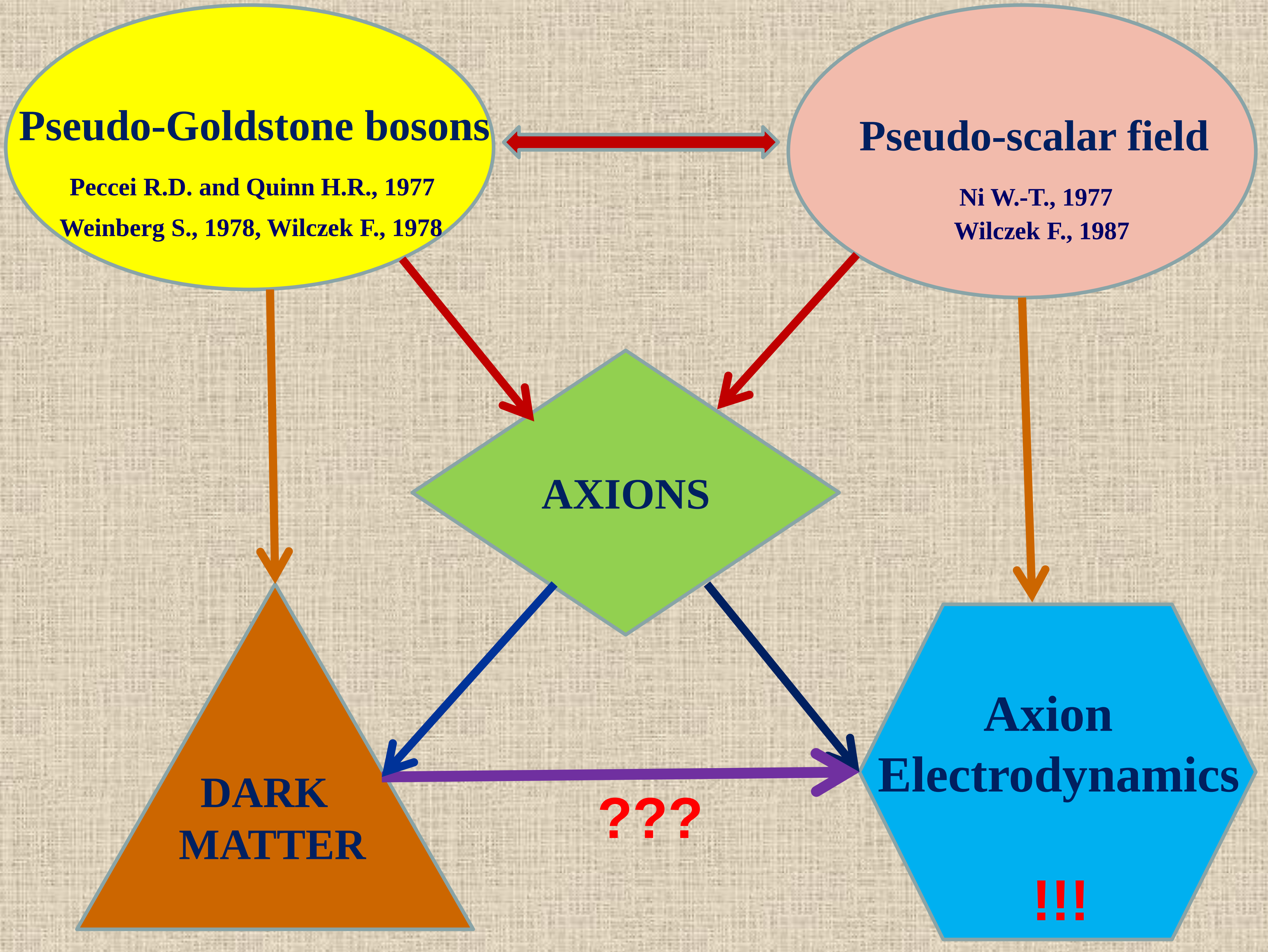
Alexander Balakin

*Department of General Relativity and Gravitation,
Institute of Physics, Kazan Federal University, Russia*

E-mail: Alexander.Balakin@ksu.ru

Items to be discussed

- **Gradient-type extension of the Einstein-Maxwell-axion model**
- **Non-minimal extension of the Einstein-Maxwell-axion model**
- **Axionic dark matter as a medium for electromagnetic waves**
- **Cosmological relic axions and dark matter**
- **Terrestrial electromagnetic field in the axionic dark matter environment**


Pseudo-Goldstone bosons
Peccei R.D. and Quinn H.R., 1977
Weinberg S., 1978, Wilczek F., 1978
Pseudo-scalar field
Ni W.-T., 1977
Wilczek F., 1987
AXIONS
DARK MATTER
???
Axion Electrodynamics
!!!

Lagrange approach to the description of the axion-photon coupling

Einstein-Maxwell-axion model (EMA)

Standard cross-term

$$S_{(0)}=\int d^4x\sqrt{-g}\left\{\frac{R+2\Lambda}{\kappa}+\frac{1}{2}F^{mn}F_{mn}+\frac{1}{2}\phi F^{mn}F^*_{mn}+ -\Psi_0^2\left[g^{mn}\nabla_m\phi\nabla_n\phi-V(\phi^2)\right]\right\}$$

Evolution equations of the gravity field

$$R_{ik}-\frac{1}{2}Rg_{ik}=\kappa\left[T^{(\mathrm{EM})}_{ik}+T^{(\mathrm{A})}_{ik}\right]$$

$$T^{(\mathrm{EM})}_{ik}\equiv\frac{1}{4}g_{ik}F^{mn}F_{mn}-F_{im}F_k{}^m$$

Stress-energy tensor of the axion field

$$T^{(\mathrm{A})}_{ik}\equiv\Psi_0^2\left\{\nabla_i\phi\nabla_k\phi-\frac{1}{2}g_{ik}\left[\nabla^m\phi\nabla_m\phi-V(\phi^2)\right]\right\}$$

There are NO cross-terms in the gravity field equations!!!

Evolution equation for the pseudoscalar (axion) field in the EMA model

$$\left[\nabla^k \nabla_k + V'(\phi^2)\right] \phi = -\frac{1}{4\Psi_0^2} F^{*mn} F_{mn}$$

Electromagnetic source of the axion field

Equations of axion electrodynamics in the EMA model

$$\nabla_k \left[F^{ik} + \phi F^{*ik}\right] = 0 \qquad \nabla_k F^{*ik} = 0$$

$$\nabla_k F^{ik} = -F^{*ik} \nabla_k \phi$$

new effects when gradient pseudo-vector is non-vanishing

$$\nabla_k \phi \neq 0$$

Applications:

Time-like	*Space-like*	*Null*
$\nabla_k \phi \nabla^k \phi > 0$	$\nabla_k \phi \nabla^k \phi < 0$	$\nabla_k \phi \nabla^k \phi = 0$
Cosmology	*Astrophysics*	*Plane-waves*

Gradient-type extensions of the Einstein-Maxwell-axion model

$$S=\int d^4x\sqrt{-g}\left\{\frac{R+2\Lambda}{\kappa}+\frac{1}{2}\left[1+\lambda_1\nabla^k\phi\nabla_k\phi\right]F^{mn}F_{mn}+\frac{\lambda_2}{2}F^{mi}\nabla_i\phi F_{mk}\nabla^k\phi+\right.$$
$$\left.+\frac{1}{2}\left[\phi+\nu U^k\nabla_k\phi\right]F^{mn}F^*_{mn}-\Psi_0^2\left[g^{mn}\nabla_m\phi\nabla_n\phi-V(\phi^2)\right]\right\}$$

We use an analogy

$\nabla_k\phi \Longleftrightarrow U_k$

$$S_{(\mathrm{m})}=\frac{1}{2}\int d^4x\sqrt{-g}\left[\frac{1}{\mu}F^{mn}F_{mn}+2\left(\varepsilon-\frac{1}{\mu}\right)F^{mi}U_iF_{mk}U^k+\phi F^{mn}F^*_{mn}\right]$$

with the action for the electromagnetic field in a medium, which is associated with the constitutive equations

$$\vec{D}=\varepsilon\vec{E}+\phi\vec{B} \qquad \vec{H}=\frac{1}{\mu}\vec{B}-\phi\vec{E}$$

δA_i → **Electromagnetic field evolution** → **New results**

$\delta\phi$ → **Evolution of axionic dark matter** → **New results**

δg_{ik} → **Gravity field evolution** → **New results**

Anisotropic Bianchi-I model

$$ds^2=dt^2-a^2(t)(dx^1)^2-b^2(t)(dx^2)^2-c^2(t)(dx^3)^2$$

Isotropic FLRW model

$$a=b=c$$

Relic cosmological axions and axionic dark matter

Two representations of the stress-energy tensor of axions

$$T_{ik}^{(A)}$$

in terms of pseudo-scalar field

$$\Psi_0^2 \left\{ \nabla_i \phi \nabla_k \phi - \frac{1}{2} g_{ik} \left[\nabla^m \phi \nabla_m \phi - V(\phi^2) \right] \right\}$$

$$=$$

in terms of dark fluid

$$W U_i U_k + q_i U_k + q_k U_i + \mathcal{P}_{ik}$$

$$\mathcal{D}\phi = \dot{\phi}$$

$$\overset{\perp}{\nabla}_i \phi = 0$$

spatial gradient of the axionic field is vanishing

$$\dot{\phi}^2 = \frac{1}{\Psi_0^2} \left[W(t) + \mathcal{P}(t) \right]$$

energy density of the axionic dark matter $W(t)$

pressure of the axionic dark matter $\mathcal{P}(t)$

Cold dark matter

$$\mathcal{P} = 0$$

$$W = \rho c^2$$

$$\dot{\phi}^2 = \frac{\rho c^2}{\Psi_0^2}$$

???

$$\rho_{(DM)} \simeq 1.25 \ \ \mathrm{GeV} \cdot \mathrm{cm}^{-3}$$

$$\frac{1}{\Psi_0} = \rho_{A\gamma\gamma}$$

$$\rho_{A\gamma\gamma} \simeq 10^{-9} \mathrm{GeV}^{-1}$$

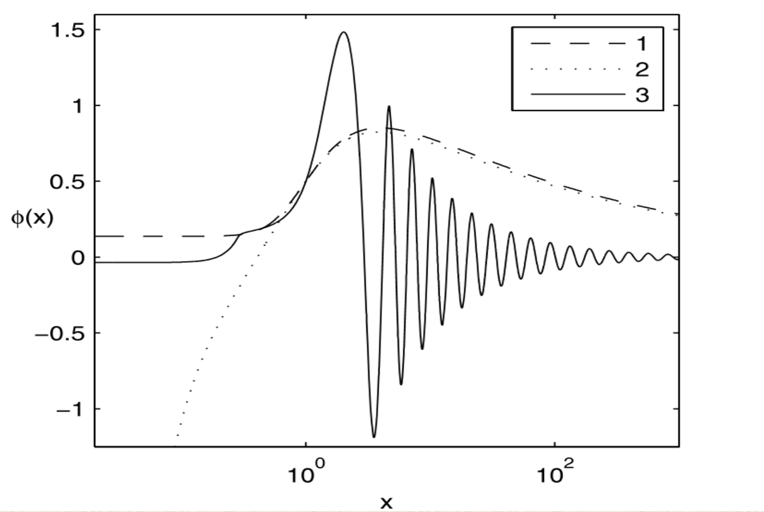
Example of the pseudo-scalar (axion) field evolution (Bianchi-I model with magnetic field)

A.B. Balakin, V.V. Bochkarev, N.O. Tarasova. EPJC, 72 (2012).

Key equation for the axion field evolution

$$\frac{1}{abc}\frac{d}{dt}\left[abc\dot{\phi}\left(\Psi_0^2 - \frac{\lambda_1 F_{12}^2}{a^2b^2}\right)\right] = \phi\left(\frac{F_{12}^2}{a^2b^2} - \Psi_0^2 V'\right)$$

Numerical simulation of the solutions for various coupling parameters



Anomalous growth of the axion number in the early Universe. Relic axions =?=dark matter

Possible scheme of the relic axion production:

1) Initial magnetic field plus axions produce cosmic electric field

$$B(t) \equiv \sqrt{F_{12}F^{12}} = \frac{F_{12}}{a(t)b(t)} \rightarrow E(t) \equiv \sqrt{-F_{30}F^{30}} = \frac{F_{12}\phi(t)}{a(t)b(t)} = B(t)\phi(t)$$

2) The source $\vec{E}\cdot\vec{B} = \phi B^2$ **produces inflation-type growth of the axion number**

Axionically induced electrodynamic effects in the GEMA model

Maxwell equations

$$\nabla_k H^{ik} = 0$$

$$H^{ik} = F^{ik} + \phi F^{*ik} + \lambda_1 F^{ik} \nabla_q \phi \nabla^q \phi + \lambda_2 \nabla^{[k} \phi F^{i]q} \nabla_q \phi$$

Dielectric permittivity tensor

$$\varepsilon^{im} = \Delta^{im}\left[1 + \lambda_1 \nabla_q \phi \nabla^q \phi\right] + \frac{1}{2}\lambda_2 \left[\Delta^{im} (\mathcal{D}\phi)^2 + \overset{\perp}{\nabla}{}^i \phi \overset{\perp}{\nabla}{}^m \phi\right]$$

$$\Delta^i_k = \delta^i_k - U^i U_k$$

projector

Magnetic impermeability tensor

$$(\mu^{-1})_{im} = \Delta_{im}\left[1 + \lambda_1 \nabla_q \phi \nabla^q \phi\right] + \frac{1}{2}\lambda_2 \left[\Delta_{im} \overset{\perp}{\nabla}_q \phi \overset{\perp}{\nabla}{}^q \phi - \overset{\perp}{\nabla}_i \phi \overset{\perp}{\nabla}_m \phi\right]$$

Tensor of magneto-electric cross-effects

$$\nu^{pm} = -\left(\phi + \nu \mathcal{D}\phi\right) \Delta^{pm} + \frac{1}{2}\lambda_2 \mathcal{D}\phi \; \epsilon^{pmkl} U_l \nabla_k \phi$$

Optical activity

Birefringence

Delay of response

induced by axions

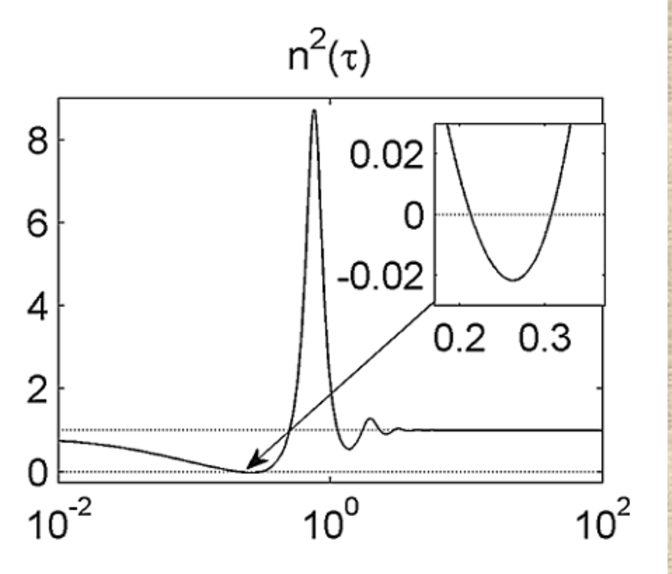
Cosmic axion electrodynamics (1)

Effective refraction index of the axionic dark matter

$$n^2(t) = \varepsilon(t)\mu(t) = \frac{1 + (\lambda_1 + \frac{1}{2}\lambda_2)\dot{\phi}^2}{1 + \lambda_1 \dot{\phi}^2}.$$

$$\varepsilon(t) = 1 + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\dot{\phi}^2 \qquad \frac{1}{\mu(t)} = 1 + \lambda_1 \dot{\phi}^2$$

Numerical simulation of the squared refractive index for various coupling parameters, and illustration of the so-called *unlighted epochs* with $n^2 < 0$ (one example; see Balakin et. al., PRD, 85 (2012) for more detail)

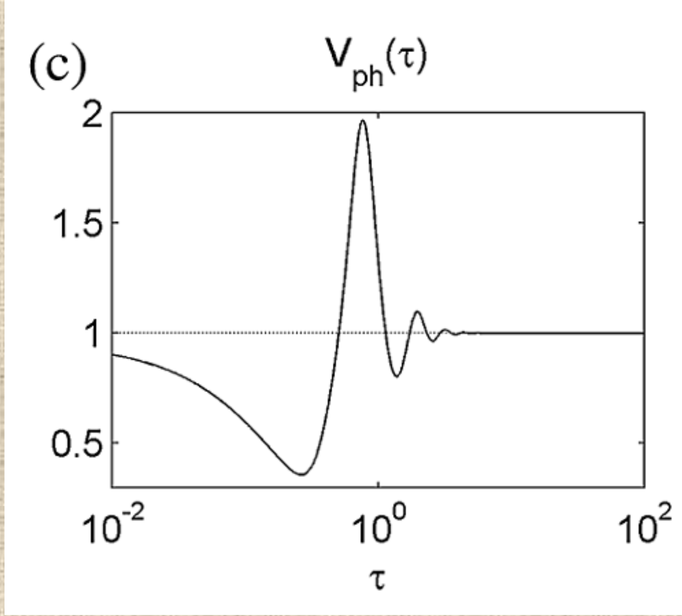


Axionic dark matter can be considered as an effective medium for the electromagnetic waves, the squared refractive index of which is linear in the dark matter particle number density

Cosmic axion electrodynamics (2)

Phase velocity of the electromagnetic waves

$$V_{\rm ph} = \frac{1}{n(t)}$$

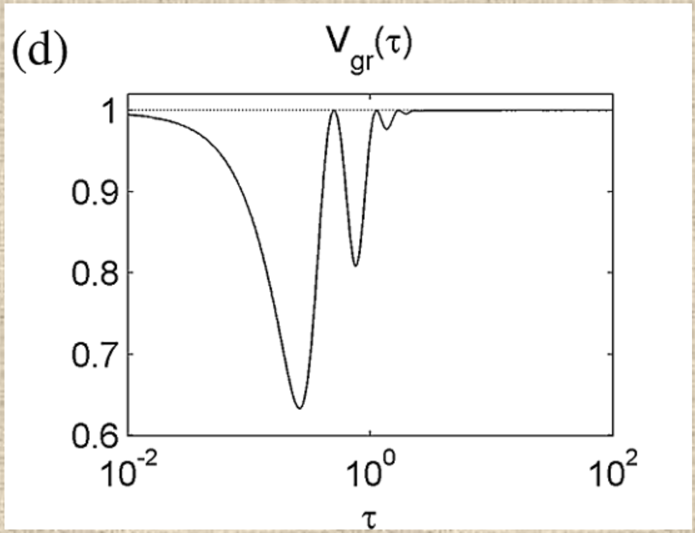


$c = 1$

Tend to the speed of light at present time, BUT depend essentially on the parameters of the axion - photon coupling in the early Universe

Group velocity of the electromagnetic waves

$$V_{\rm gr} = \frac{2n}{1+n^2}$$



Is it necessary to RE-estimate the cosmic distances measured by optical devices ???

Cosmic axion electrodynamics (3)

A.B. Balakin, N.O. Tarasova. Gravitation and Cosmology, 18, 2012.

Gradient-type extension of the EMA model predicts
a non-stationary polarization rotation of the electromagnetic waves
(non-stationary optical activity induced by axionic dark matter)

Example of exact solution of the equations of *axion electrodynamics* for the isotropic FLRW model

$$A_2 = -A_0 \sin\left[W - \varphi(t)\right], \quad A_3 = A_0 \cos[W - \varphi(t)],$$

Basic phase of the electromagnetic wave

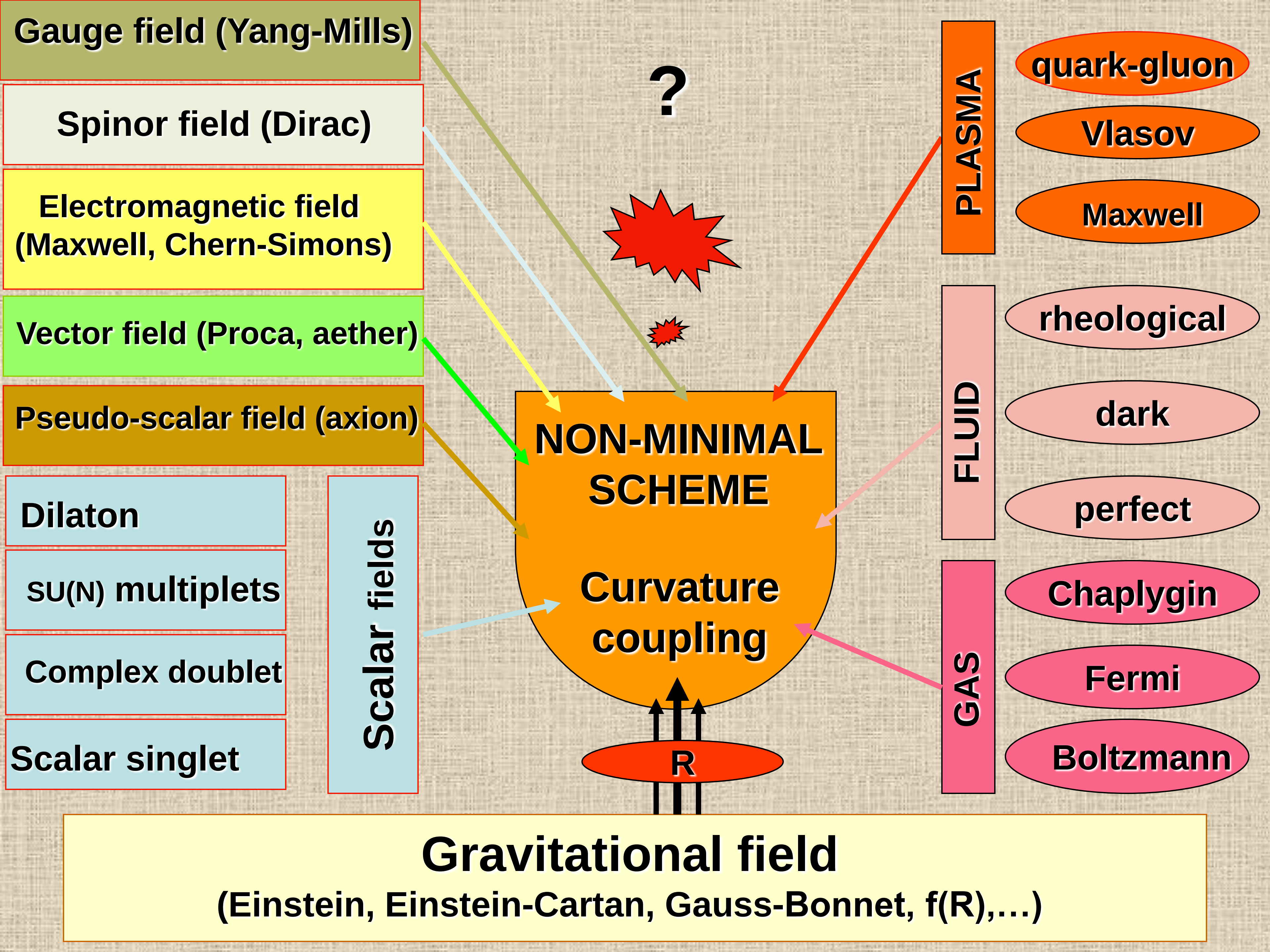
$$W = W(t_0) + k\left[\int_{t_0}^{t} \frac{dt'}{a(t')} - x\right]$$

Axionically induced phase variation

$$\varphi(t) \equiv \Theta(t) - \Theta(t_0)$$

Angle of the non-stationary rotation

$$\Theta(t) \equiv \frac{1}{2}\left[\phi(t) + \nu\dot{\phi}(t)\right]$$



Gauge field (Yang-Mills)
Spinor field (Dirac)
Electromagnetic field (Maxwell, Chern-Simons)
Vector field (Proca, aether)
Pseudo-scalar field (axion)
Dilaton
SU(N) multiplets
Complex doublet
Scalar singlet
Scalar fields
?
NON-MINIMAL SCHEME
Curvature coupling
R
PLASMA
quark-gluon
Vlasov
Maxwell
FLUID
rheological
dark
perfect
GAS
Chaplygin
Fermi
Boltzmann
Gravitational field
(Einstein, Einstein-Cartan, Gauss-Bonnet, f(R),…)

Non-minimal extension of the EMA model

A.B. Balakin and Wei-Tou Ni, Class. Quantum Grav., 27 (2010)

$$S_{(\mathrm{NM})} = \int \mathrm{d}^4 x \sqrt{-g}\, \left\{ \tfrac{1}{2} \mathcal{R}^{ikmn} F_{ik} F_{mn} + \tfrac{1}{2} \chi_{(\mathrm{A})}^{ikmn} \phi F_{ik} F^{*}_{mn} - \right.$$

$$\left. - \Re^{mn}_{(\mathrm{A})} \nabla_m \phi \nabla_n \phi + \eta_{(\mathrm{A})} R \phi^2 \right\}$$

Non-minimal susceptibility tensors and coupling constants

$$\mathcal{R}^{ikmn} = q_1 R g^{ikmn} + q_2 \Re^{ikmn} + q_3 R^{ikmn}$$

$$\chi_{(\mathrm{A})}^{ikmn} = Q_1 R g^{ikmn} + Q_2 \Re^{ikmn} + Q_3 R^{ikmn}$$

$$\Re^{mn}_{(\mathrm{A})} \equiv \tfrac{1}{2} \eta_1 \left(F^{ml} R^{n}{}_{l} + F^{nl} R^{m}{}_{l} \right) + \eta_2 R g^{mn} + \eta_3 R^{mn}$$

$$g^{ikmn} \equiv \tfrac{1}{2} (g^{im} g^{kn} - g^{in} g^{km})$$

$$\Re^{ikmn} \equiv \tfrac{1}{2} (R^{im} g^{kn} - R^{in} g^{km} + R^{kn} g^{im} - R^{km} g^{in})$$

Example: regular plane-wave (gravitational wave solution) supported by the axionic field (axionic dark matter?)

$$ds^2 = 2\,du\,dv - \{\exp[2\beta_{(\max)}\sin^2\lambda u](dx^2)^2 + \exp[-2\beta_{(\max)}\sin^2\lambda u](dx^3)^2\}$$

$$\det(g_{ik}) = -1 \neq 0$$ **(regular)**

Axionically induced polarization rotation produced by space-time curvature in case of *constant* (!) pseudo-scalar field

$$A_2 = B_0 e^{\beta}\cos(Q_3\phi_0\tau)\sin W \qquad A_3 = -B_0 e^{-\beta}\sin(Q_3\phi_0\tau)\cos W$$

$$\frac{(A_2\,e^{-\beta}/B_0)^2}{\cos^2(Q_3\phi_0\tau)} + \frac{(A_3\,e^{\beta}/B_0)^2}{\sin^2(Q_3\phi_0\tau)} = 1$$

Axion dark matter fingerprints in the terrestrial magnetic and electric fields

A.B. Balakin and L.V. Grunskaya. Reports on Mathematical Physics, 71, 2013.

Prologue

$$\mathrm{rot}\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}, \qquad \mathrm{div}\vec{E} = 4\pi\rho - \vec{B}\cdot\vec{\nabla}\phi,$$

$$\mathrm{div}\vec{B} = 0, \qquad \mathrm{rot}\vec{B} = \frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \frac{4\pi}{c}\vec{J} + \frac{1}{c}\vec{B}\frac{\partial}{\partial t}\phi - \vec{E}\times\vec{\nabla}\phi$$

Axion electrodynamics in terms of three-vectors (e.g., Wilczek F., PRL,1987)

Exact solution exists for $\phi = \phi(t)$

$$0 = \frac{d}{dt}\vec{E} + \vec{B}\frac{d}{dt}\phi \;\longrightarrow\; \vec{B} = \vec{B}_0 = \mathrm{const}$$

$$\vec{E}(t) = \vec{E}(0) - [\phi(t) - \phi(0)]\vec{B}_0$$

Axionically produced *Longitudinal E-B Cluster*

When magnetic field lines are curved, exact solutions demonstrate the following features:

Terrestrial magnetic field distortion by the relic axions

Static exact solution in EMA model

$$B_{(\mathrm{rad})}(r,q) = -\frac{2\mu}{r^3}\cos\theta\,(\cos qr + qr\sin qr)\,,$$

$$B_{(\mathrm{merid})}(r,q) = \frac{\mu\sin\theta}{r^3}\left[(\cos qr + qr\sin qr) - q^2r^2\cos qr\right]$$

$$B_{(\mathrm{azim})}(r,q) = -q\sin\theta\frac{\mu}{r^2}(\cos qr + qr\sin qr)\,.$$

$$q \equiv \frac{d\phi}{dx^0} = \frac{a}{c}\dot{\phi}$$

$$q = \pm\frac{a}{\Psi_0}\sqrt{W+P}$$

When the axion field can be treated as negligible, we obtain the standard dipole-type representation of the terrestrial magnetic field

$$B_{(\mathrm{rad})}(r,0) = -\frac{2\mu}{r^3}\cos\theta\,,$$

$$B_{(\mathrm{merid})}(r,0) = \frac{\mu\sin\theta}{r^3}$$

$$B_{(\mathrm{azim})}(r,0) = 0$$

Terrestrial magnetic field as a function of altitude is non-monotonic due to the axion environment ???

$$\cos[qR^*_{(m)}] + qR^*_{(m)}\sin[qR^*_{(m)}] = 0$$

At $r = R^*_{(m)}$ **the radial and azimuthal components change the signs**

? Visual drift of the Earth magnetic pole ?

$$\tan\delta \equiv \frac{B_{(\mathrm{azim})}(R)}{B_{(\mathrm{merid})}(R)} \rightarrow -qR \rightarrow 10^{-7}$$

$$g_{A\gamma\gamma} \simeq 10^{-9}\mathrm{GeV}^{-1} \qquad \rho_{(\mathrm{DM})} \simeq 0.033\ M_{(\mathrm{Sun})}\mathrm{pc}^{-3}$$

Oscillations in the resonator Earth-Ionosphere

Axionic modifications of the equations for Debye potentials

$$\Delta_{(1)} V - \frac{\partial^2}{\partial x^{02}} V = q \frac{\partial}{\partial x^0} U \qquad \Delta_{(1)} U - \frac{\partial^2}{\partial x^{02}} U = -q^2 U - q \frac{\partial}{\partial x^0} V$$

Example of exact solution of the boundary value problem:
perturbations of *meridional* electric field produce variations of *meridional* magnetic field in the axion dark matter environment (and vice versa)

The simplest initial conditions: $v_{nj}(0) = 0$ $\quad \dot{v}_{nj}(0) \neq 0$ $\quad u_{nj}(0) = 0$ $\quad \dot{u}_{nj}(0) = 0$

Mode amplitude for *meridional* electric perturbations

$$v_{nj}(\tilde{t}) = \frac{\dot{v}_{nj}(0)}{\omega_{0nj}} \sin \omega_{0nj} \tilde{t}$$

Mode amplitude for axionically induced *meridional* magnetic variations

$$u_{nj}(\tilde{t}) = \frac{1}{2} q c \tilde{t} \; v_{nj}(\tilde{t})$$

An example of solution describing the so-called *non-stationary Longitudinal Clusters* in the terrestrial electromagnetic field !!!

Outlook

Relic axions forming (?) dark matter, produce oscillations of a new type in the terrestrial electrodynamic system, which belong to the class of the so-called Longitudinal Electro – Magnetic Clusters. We deal with correlated variations of the electric and magnetic fields, which are parallel to one another (e.g., radial-radial or azimuthal-azimuthal) and are coupled by the axion field !

Correlation analysis of infra-low-frequency variations of the terrestrial electric and magnetic fields (i.e., variations with the frequency in the range $10^{-5} - 10^{-3}$ Hz **) - is the basic idea of experiments aimed on detection of specific Longitudinal Electro-Magnetic Clusters in the resonator Earth-Ionosphere (Team from University of Vladimir, Russia).**
In case of success one could speak about indirect finding of fingerprints of the relic axions in the terrestrial electric and magnetic fields. First experimental results are planned to be presented at GR20 Conference in Warsaw (7-13 July, 2013).

THANK FOR YOUR ATTENTION !