Propagation of axions and photons in a cold axion condensate

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Introduction

Cold relic axions resulting from vacuum misalignment in the early universe is a popular and viable candidate to dark matter.

Provided that the reheating temperature after inflation is below the Peccei-Quinn transition scale, in later times the axion evolves as

$$a(t) = a_0 \sin m_a t, \quad \mathbf{k} = 0 \qquad \rho \simeq a_0^2 m_a^2$$

$$\rho \simeq 10^{-10}\mathrm{eV}^4, \quad \rho^* \simeq 10^{-4}\mathrm{eV}^4 \quad (30 \text{ to } 100 \text{ kpc})$$

The axion background provides a very diffuse concentration of a pseudoscalar condensate that affects the propagation of particles coupled to it such as photons. Can it be detected ?

In this talk I will discuss several non-standard effects that *might* help.

Outline

- Introduction
- Propagation in a cold axion background
- Three physical effects
    1. Momentum gaps: some photon wavelengths cannot exist
    2. Magnetic field in a cold condensate: changing some characteristics of the Primakoff effect
    3. Bouncing off the axion wall: trapped photons
- Conclusions and outlook

Propagation of photons in a cold axion background

Let us consider electromagnetism in a background where Lorentz symmetry is broken by means of a time-like vector

$$\mathcal{L} = \mathcal{L}_{\rm INV} + \mathcal{L}_{\rm LIV}$$

$$\mathcal{L}_{\rm INV} = -\,\frac{1}{4}\,F^{\alpha\beta}F_{\alpha\beta} \qquad \mathcal{L}_{\rm LIV} = \frac{1}{2}\,m_V^2\,A_\mu\,A^\mu + \,\frac{1}{2}\,\eta_\alpha A_\beta \widetilde{F}^{\,\alpha\beta}$$

E.o.M.:

$$\left\{g^{\,\lambda\nu}\left(k^2 - m_V^2\right) + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\eta_\alpha\,k_\beta\right\}\tilde{A}_\lambda(k) = 0$$

We can build two complex and space-like chiral polarization vectors $\varepsilon_\pm^\mu(k)$ which satisfy the orthonormality relations

$$-\,g_{\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\pm^{\nu}(k) = 1 \qquad g_{\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\mp^{\nu}(k) = 0$$

In addition we have

$$\varepsilon_T^\mu(k) \sim k^\mu \qquad \varepsilon_L^\mu(k) \sim k^2\eta^\mu - k^\mu\eta\cdot k$$

$$g_{\mu\nu}\,\varepsilon_A^{\mu\,*}(k)\,\varepsilon_B^{\nu}(k) \,=\, g_{AB} \qquad g^{\,AB}\,\varepsilon_A^{\mu\,*}(k)\,\varepsilon_B^{\nu}(k) \,=\, g^{\,\mu\nu}$$

Propagation of photons in a cold axion background

Let us now assume that $\eta_\alpha = \partial_\alpha a(t) = \eta\delta_{\alpha 0}$

The polarization vectors of positive and negative chirality are solutions of the vector field equations if and only if

$$k^\mu_\pm = (\omega_{\mathbf{k}\,\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\,\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta|\mathbf{k}|}$$

In order to avoid problems with causality we want $k_\pm^2 \geq 0$.
Photons of positive chirality have no problems with causality
Photons of negative chirality exist as asymptotic states iff

$$|\mathbf{k}| < \frac{m_V^2}{\eta}$$

For $m_V = 0$ they cannot exist as asymptotic states. Changing $\eta \to -\eta$ exchanges the chirality of photons.

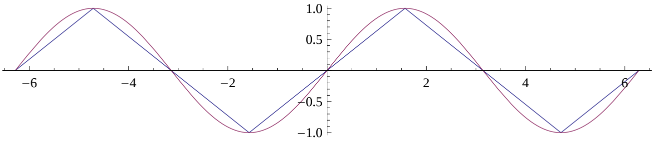
Propagation of photons in a cold axion background

Axion-photon coupling:

$$\Delta\mathcal{L} = -g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0}{f_a}\cos(m_a t)\,\epsilon^{ijk}A_i F_{jk}$$

where $\partial_\alpha a(t) = (\eta(t), 0, 0, 0)$, and $\eta(t) = \eta_0 \cos m_a t$. Popular models such as DFSZ and KSVZ all give $g_{a\gamma\gamma} \simeq 1$.

If momenta are large $\mathbf{k} >> m_a$ it makes sense to treat the axion background adiabatically with a (quasiconstant) derivative



It makes sense to approximate the sinusoidal variation piecewise by a square profile $\eta(t) = \pm\eta_0$ with period $2\pi/m_a$

Propagation of photons in a cold axion background

Astrophysical bounds:

$$f_a > \mathcal{O}(10^{10})\ \mathrm{GeV}, \qquad 10^{-2}\mathrm{eV} > m_a > 10^{-6}\ \mathrm{eV}$$

$$\Rightarrow |\eta| \simeq \alpha \frac{\sqrt{\rho_a}}{f_a} \simeq 10^{-23} - 10^{-24}\mathrm{eV}$$

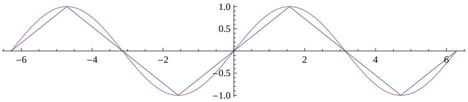
Direct bounds are weaker:

$$|\eta| < 10^{-20}\mathrm{eV}$$

η is the relevant quantity for all the effects discussed in this talk.

All the considerations in this presentation refer to vacuum propagation, i.e. we take $m_\gamma = 0$. Everything is computed at tree level in QED but non-linearities such as the ones described by the Euler-Heisenberg effective lagrangian could be included.

$a(t)$ changes sign with a period $2\pi/m_a$. Let us approximate the sinusoidal variation and solve exactly for the propagating modes



The equation for $\hat{A}_\nu(t,\vec{k})$ is

$$\left[g^{\mu\nu}(\partial_t^2+\vec{k}^2)-i\epsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta\right]\hat{A}_\nu(t,\vec{k})=0$$

$$\hat{A}_\nu(t,\vec{k})=\sum_{\lambda=+,-} f_\lambda(t)\varepsilon_\nu(\vec{k},\lambda)$$

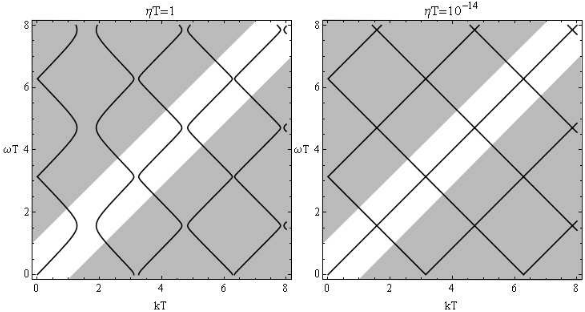
We write $f(t)=e^{-i\omega t}g(t)$ and demand that $g(t)$ have the same periodicity as $\eta(t)$. This requires

$$\cos(2\omega T)=\cos(\alpha T)\cos(\beta T)-\frac{\alpha^2+\beta^2}{2\alpha\beta}\sin(\alpha T)\sin(\beta T),\qquad T=\frac{\pi}{M_a},$$

Forbidden wavelengths

As η_0/m_a grows there is a surprise ($\eta_0 = 2g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0 m}{f_a}$)

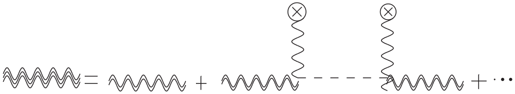
$$\cos(2\omega T) = \cos(\alpha T)\cos(\beta T) - \frac{\alpha^2+\beta^2}{2\alpha\beta}\sin(\alpha T)\sin(\beta T)$$



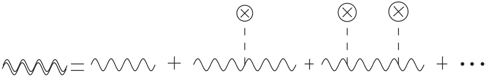
Some photon wavelengths are forbidden in the universe if there is a cold axion background.
Can this be seen in table-top experiments?

Adding a magnetic field

If $\eta_0 = 0$ the theoretical technology is well known. Used to analyze the results of CAST, ADMX, ALPS ....



Interaction with the cold axion background implies that we need to take into account



Relevant parameters

$$b = 2g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{B}{f_a} \qquad \eta_0 = 2g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0 m}{f_a}$$

assuming $f_a = 10^7$ GeV

$$B = 10\ \mathrm{T} \Rightarrow b \leq 10^{-15}\,\mathrm{eV} \qquad \eta_0 \leq 10^{-20}\,\mathrm{eV}$$

Adding a magnetic field

The ratio b/η_0 is governed by the ratio $B/a_0 m_a$. Observationally $\eta_0 << b$ so it makes sense to treat η as a perturbation on the solution with $b \neq 0$

$$\left(\begin{array}{ccc} \partial_t^2 + k^2 + m^2 & -ib\partial_t & 0 \\ -ib\partial_t & \partial_t^2 + k^2 & \eta k \\ 0 & \eta k & \partial_t^2 + k^2 \end{array} \right) \left(\begin{array}{c} \hat{a} \\ i\hat{A}_1 \\ \hat{A}_2 \end{array} \right) = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right)$$

We try a solution of the form: $(\hat{a}, i\hat{A}_1, \hat{A}_2) = e^{-i\omega t}(x, X_1, X_2)$:

$$\left(\begin{array}{ccc} -\omega^2 + k^2 + m^2 & \omega b & 0 \\ \omega b & -\omega^2 + k^2 & \eta k \\ 0 & \eta k & -\omega^2 + k^2 \end{array} \right) \left(\begin{array}{c} x \\ X_1 \\ X_2 \end{array} \right) = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right)$$

Reduced magnetic field: $\vec{b} = (b, 0, 0)$

Adding a magnetic field

Cold axion background oscillations change $\eta \leftrightarrow -\eta$ with periodicity $\sim 1/m_a$

Within each period the proper frequencies are $(\eta \ll b \ll \{m, k\})$

$$\begin{aligned} \omega_a^2 &\approx k^2 + m^2 + b^2 + \frac{b^2 k^2}{m^2} + \frac{b^2 \eta^2 k^4}{m^6} + \frac{b^2 \eta^2 k^2}{m^4} \\ \omega_1^2 &\approx k^2 - \frac{b^2 \eta^2 k^4}{m^6} - \frac{b^2 \eta^2 k^2}{m^4} - \frac{b^2 k^2}{m^2} - \frac{\eta^2 m^2}{b^2} \\ \omega_2^2 &\approx k^2 + \frac{\eta^2 m^2}{b^2} \end{aligned}$$

To be compared with

$$w_\pm^2 = k^2 \pm \eta k$$

(NB: do not attempt to take the $b \to 0$ limit in the general case)

Adding a magnetic field

Numerically for $b = 10^{-15}$ eV $(10\,T)$, $\eta = 10^{-24}$ eV and $k = 1$ keV, the most relevant terms are

$$\begin{aligned}
\omega_a^2 &\approx k^2 + m^2 + \frac{b^2 k^2}{m^2} \\
\omega_1^2 &\approx k^2 - \frac{b^2 k^2}{m^2} - \frac{\eta^2 m^2}{b^2} \\
\omega_2^2 &\approx k^2 + \frac{\eta^2 m^2}{b^2}
\end{aligned}$$

The splitting between the two polarizations goes as

$$\omega_1^2 - \omega_2^2 \approx -\frac{b^2 k^2}{m^2} - 2\frac{\eta^2 m^2}{b^2}$$

The relevance of the CAB is somehow enhanced by the magnetic field and dominates for

$$b < m_a \sqrt{\frac{\eta}{k}} \simeq 10^{-14} m_a$$

Adding a magnetic field

There is also a change in the plane of polarization

$$\sim \frac{\eta m^2}{b^2 k}$$

(for "large" axion masses, the actual result is more complicated)
The angle can be as large as 10^{-3}

Recall the modification to QED brought about by an axion-like background

$$\Delta\mathcal{L} = \frac{1}{2}\eta_\alpha A_\beta \tilde{F}^{\alpha\beta}$$

This piece changes slightly the dispersion relation of photons. We will now explore different possible axion backgrounds (other than the cold background oscillating in time with period $\sim 1/m_a$)

$$-\tfrac{1}{4}\, F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\,\zeta_\lambda x^\lambda\,\theta(-\zeta\cdot x) \leftrightarrow \tfrac{1}{2}\,\zeta_\mu A_\nu(x)\widetilde{F}^{\mu\nu}(x)\,\theta(-\zeta\cdot x),$$

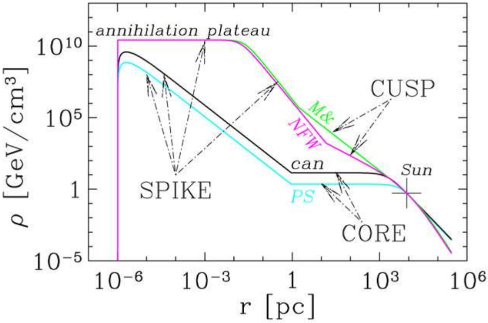
This associates a space-like boundary with a space-like CS vector

$$\zeta_\mu = \zeta \times (0, \vec{a}) \quad |\vec{a}| = 1$$

(LIV vector renamed from η_μ to ζ_μ to avoid confusion with CAB)

Crossing the boundary

Compact dense stars filled by axions with density degrading to their surface?
Galactic dark matter profiles?

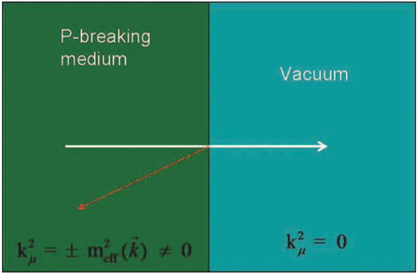


Axial chemical potential for a fireball in heavy ion collisions (scales completely different!)?

Crossing the boundary

Even if not totally realistic let us use a linearly varying background (it can be solved easily)

For simplicity let us place the boundary "wall" in the $\hat{X}$ direction



Matching on the boundary $\zeta \cdot x = 0$

$$\delta(\zeta \cdot x)\left[\, A^\mu_{\rm vacuum}(x) - A^\mu_{\rm CS}(x) \,\right] = 0$$

Crossing the boundary

Abnormal dispersion laws for different polarizations in the parity broken phase

$$\begin{cases} k_{1L} = k_{10} = \sqrt{\omega^2 - m^2 - k_\perp^2} \\ k_{1+} = \sqrt{\omega^2 - m^2 - k_\perp^2 + \zeta\sqrt{\omega^2 - k_\perp^2}} \\ k_{1-} = \sqrt{\omega^2 - m^2 - k_\perp^2 - \zeta\sqrt{\omega^2 - k_\perp^2}} \end{cases}$$

Usual dispersion law in the normal phase

$$k_1 = \sqrt{\omega^2 - m^2 - k_\perp^2}$$

Different dispersion relations lead to non-trivial reflection and transmision coefficients

Crossing the boundary

$$M^2 \equiv k_\mu k^\mu = m^2 - \zeta\sqrt{\omega^2 - k_\perp^2}$$

$$k_{1L} = \sqrt{\frac{(M^2 - m^2)^2}{\zeta^2} - m^2} \qquad k_{1\pm} = \sqrt{\frac{(M^2 - m^2)^2}{\zeta^2} - M^2}$$

In this notation the $\pm$ dispersion relations apparently coincide but M^2 has different domains of definition

$$M_+^2 < (\sqrt{m^2 + \frac{\zeta^2}{4}} - \frac{\zeta}{2})^2 \quad M_-^2 < (\sqrt{m^2 + \frac{\zeta^2}{4}} + \frac{\zeta}{2})^2$$

Then

$$\kappa_{ref}(M^2) = \frac{|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - M^2} - \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - m^2}|}{|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - M^2} + \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - m^2}|}$$

Crossing the boundary

Photon escaping from the axion sphere.

Recall $M^2/\zeta = m^2/\zeta^2 - \sqrt{\omega^2 - k_\perp^2}$

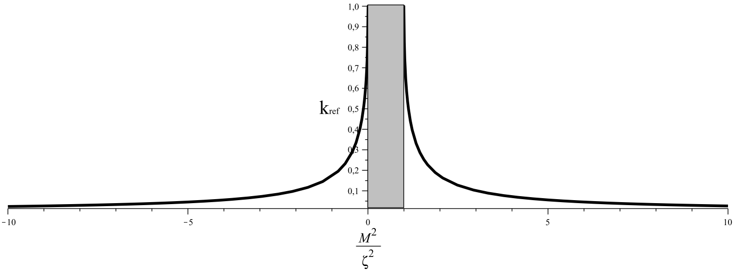


Figure: Reflection coefficient for photons ($m = 0$) escaping. The kinematically forbidden region is shaded

Crossing the boundary

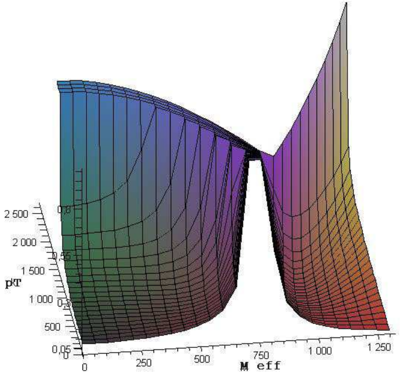


Figure: Reflection coefficient for vector mesons ($m \neq 0$escaping. The scale is the relevant one for heavy ion collisions

Crossing the boundary

In the context of axion physics the effect seems to depend crucially on a ratio of two numbers that are both small: m_V and ζ

It is also quite interesting to study to photons attempting to enter the axion-sphere.

The astrophysical consequences of the above results yet to be worked out...